VSMCOLLEGEOFENGINEERING

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Departmentof

ELECTRICAL & ELECTRONICS ENGINEERING

ELECTROMAGNETIC FIELD THEORY

SUBJECT MATERIAL

YEAR :II   SEMESTER :I

Regulaton:VR23   SubjectCode:VR2321201

Prepared by →

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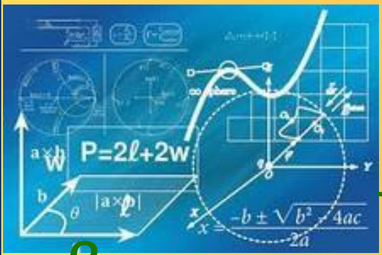
EEE Department



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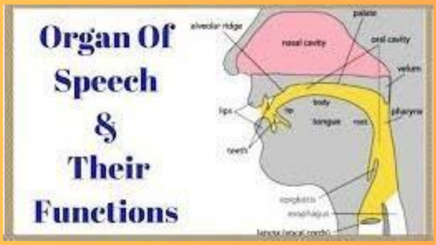

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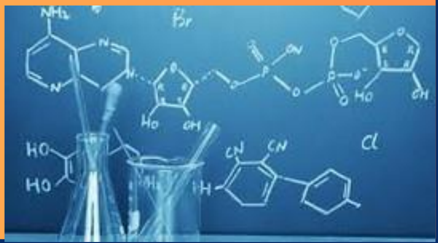

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ELECTRICAL & ELECTRONICS ENGINEERING

Subject Material

ELECTROMAGNETIC FIELD THEORY

II B.TECH I SEM

Regulation: VR23

Subject Code: VR2321201

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Ramachandrapuram-533255

II B.TECH I SEM

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SUB CODE: VR2321201

Electromagnetic Field Theory

Course Objectives:

- To study the production of electric field and potentials due to different configurations
- of static charges.
- To study the properties of conductors and dielectrics, calculate the capacitance of
- different configurations. Understand the concept of conduction and convection current
- densities.
- To study the magnetic fields produced by currents in different configurations,
- application of Ampere's law and the Maxwell's second and third equations.
- To study the magnetic force and torque through Lorentz force equation in magnetic
- field environment like conductors and other current loops.
- To develop the concept of self and mutual inductances and the energy stored.
- To study time varying and Maxwell's equations in different forms and Maxwell's fourth equation

  for the induced EMF.

Course Outcomes:

At the end of the course, student will be able to,

CO1: Compute electric fields and potentials using Gauss law/ solve Laplace's or Poisson's equations for various electric charge distributions.

CO2: Analyse the behaviour of conductors in electric fields, electric diploe and the capacitance and energy stored in dielectrics.

CO3: Calculate the magnetic field intensity due to current carrying conductor and understanding the application of Ampere's law, Maxwell's second and third law.

CO4: Estimate self and mutual inductances and the energy stored in the magnetic field.

CO5: Understand the concepts of Faraday's laws, Displacement current, Poynting theorem and Poynting vector.

UNIT - I

Vector Analysis:

Vector Algebra: Scalars and Vectors, Unit vector, Vector addition and subtraction, Position and distance vectors, Vector multiplication, Components of Coordinate Systems: Rectangular, Cylindrical and Spherical coordinate systems. Vector Calculus: Differential length, Area and Volume. Del operator, Gradient of a scalar, Divergence of a vector and Divergence theorem (definition only). Curl of a vector and Stoke's theorem (definition only), Laplacian of a scalar

Electrostatics:

Coulomb's law and Electric field intensity (EFI)- EFI due to Continuous charge distributions (line and surface charge), Electric flux density, Gauss's law (Maxwell's first equation, $\nabla.\bar{D} = p_v$), Applications of Gauss's law, Electric Potential, Work done in moving a point charge in an electrostatic field (second Maxwell's equation for static electric fields,$\nabla \times \bar{E} = 0$), Potential gradient, Laplace's and Poison's equations.

UNIT - II

Conductors – Dielectrics and Capacitance:

Behaviour of conductor in Electric field, Electric dipole and dipole moment Potential and EFI due to an electric dipole, Torque on an Electric dipole placed in an electric field, Current density-conduction and convection current densities, Ohm's law in point form, Behaviour of conductors in an electric field, Polarization, dielectric constant and strength, Continuity equation and relaxation time, Boundary conditions between conductor to dielectric, dielectric to dielectric and conductor to free space, Capacitance of parallel plate, coaxial and spherical capacitors, Energy stored and density in a static electric field.

UNIT - III

Magneto statics, Ampere's Law and Force in magnetic fields:

Biot-Savart's law and its applications viz. Straight current carrying filament, circular, square, rectangle and solenoid current carrying wire Magnetic flux density and Maxwell's second Equation ($\nabla.\vec{\bar{B}} = 0$), Ampere's circuital law and its applications viz. MFI due to an infinite sheet, long filament, solenoid, toroidal current carrying conductor, point form of Ampere's circuital law, Maxwell's third equation ($\nabla \times \bar{H} = \vec{J}$).

Magnetic force, moving charges in a magnetic field Lorentz force equation, force on a current element in a magnetic field, force on a straight and a long current carrying conductor in a magnetic field, force between two straight long and parallel current carrying conductors, Magnetic dipole, Magnetic torque, and moment.

UNIT - IV

Self and mutual inductance:

Self and mutual inductance – determination of self-inductance of a solenoid, toroid, coaxial cable and mutual inductance between a straight long wire and a square loop wire in the same plane – Energy stored and energy density in a magnetic field.

UNIT - V

Time Varying Fields:

Faraday's laws of electromagnetic induction, Maxwell's fourth equation ($\nabla\times \bar{E} = -\frac{d\bar{B}}{dt}$), integral and point forms of Maxwell's equations, statically and dynamically induced EMF,Displacement current, Modification of Maxwell's equations for time varying fields, Poynting theorem and Poynting vector.

Textbooks:

1. "Elements of Electromagnetics" by Matthew N O Sadiku, Oxford Publications,7th edition, 2018.
2. "Engineering Electromagnetics" by William H. Hayt& John. A. Buck Mc. Graw-Hill 7th Editon.2006.

Reference Books:

1. "Introduction to Electro Dynamics" by D J Griffiths, Prentice- Hall of India Pvt. Ltd, 2nd edition.
2. "Electromagnetic Field Theory" by Yaduvir Singh, Pearson India, 1st edition, 2011.
3. "Fundamentals of Engineering Electromagnetics" by Sunil Bhooshan, Oxford University Press, 2012.
4. Schaum's Outline of Electromagnetics by Joseph A. Edminister, Mahamood Navi 4th, Edition,2014.

Online Learning Resources:

1. https://archive.nptel.ac.in/courses/108/106/108106073/
2. https://nptel.ac.in/courses/117103065

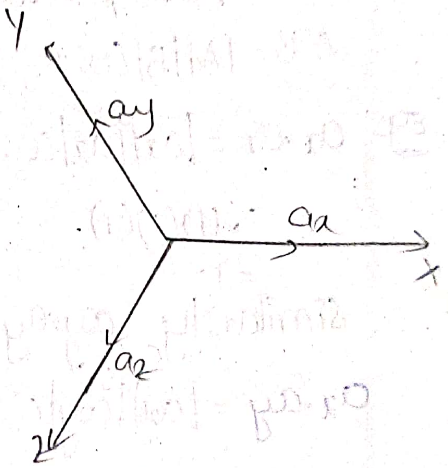
UNIT - I

Vector:-

$$\vec{A} = \bar{A}_x a_x + \bar{A}_y a_y + \bar{A}_z a_z$$

where a_x, a_y, a_z are unit vectors

i.e A_x, A_y, A_z are the co-efficients of a vector "A" in x, y, z



$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

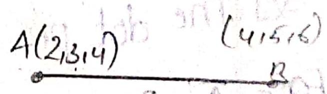
i.e unit vector a is

$$a_A = \frac{\bar{A}}{|A|}$$

$$a_A = \frac{\bar{A}_x a_x + \bar{A}_y a_y + \bar{A}_z a_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Eg:- $\bar{A} = 2a_x + 3a_y + 4a_z$; $\bar{B} = 5a_x + 4a_y + 6a_z$ Find distance between two vector.

Distance between two vector



$2a_x + 3a_y + 4a_z$,

$4a_x + 5a_y + 6a_z$

$$\bar{R}_{AB} = \bar{B} - \bar{A}$$

$$= 5a_x + 4a_y + 6a_z - 2a_x - 3a_y - 4a_z$$

$$= 3a_x + a_y + 2a_z$$

Magnitude of $\bar{R}_{AB}$:

$$|R_{AB}| = \sqrt{3^2 + 1^2 + 2^2} = \sqrt{9+1+4} = \sqrt{14}$$

unit vector:-

$$a_A = \frac{\bar{A}}{|A|}$$

$$= \frac{2a_x + 3a_y + 4a_z}{\sqrt{2^2 + 3^2 + 4^2}} = \frac{2a_x + 3a_y + 4a_z}{\sqrt{29}}$$

unit vector $a_{R_{AB}} = \dfrac{R_{AB}}{|R_{AB}|}$

$$= \frac{3a_x + a_y + 2a_z}{\sqrt{14}}$$

Vector addition and Substitraction

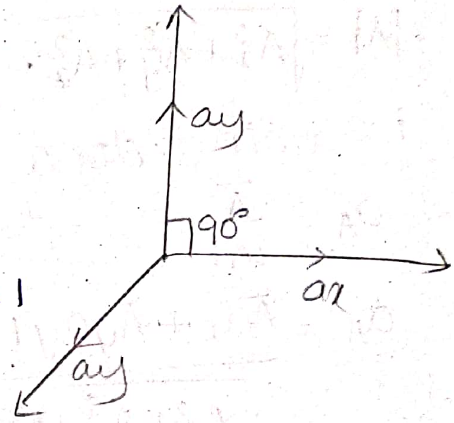
$\bar{A} + \bar{B} = \bar{B} + \bar{A}$

$\bar{A} + (\bar{B} + \bar{C}) = (\bar{A} + \bar{B}) + \bar{C}$

$\bar{A} - \bar{B} = \bar{B} - \bar{A}$

Scalar / dot product:-

$A \cdot B = |A||B| \cos\theta$



Eg:- $a_x \cdot a_x = |a_x||a_x| \cos(a_x \cdot a_x)$

$= (1)(1)(1)$

$= 1$

similarily $a_y \cdot a_y = 1$ $a_z \cdot a_z = 1$

$a_x \cdot a_y = |a_x||a_y| \cos(a_x \cdot a_y)$

$= (1)(1)(\cos 90°$

$= 0$

Similarily $a_x \cdot a_y = 0$; $a_y \cdot a_z = 0$; $a_z \cdot a_x = 0$

* The dot product is scalor product there is no direction

Eg:- $\bar{A} = 3a_x + 4a_y + 5a_z$; $\bar{B} = 6a_x + 4a_y + 2a_z$

$\bar{A} \cdot \bar{B} = (3a_x' + 4a_y + 5a_z) \cdot (6a_x + 4a_y + 2a_z)$

$= 18 + 16 + 10$

$= 44$

$\therefore \bar{A} \cdot \bar{B} = \bar{B} \cdot \bar{A}$

10-09-22

Cross product (or) vector product:-

$\bar{A} \times \bar{B} = |\bar{A}||\bar{B}| \sin\theta \, \hat{a}_n$

eg: $a_x \times a_x = |a_x||a_x| \sin 0° \hat{a}_n$

$= (1)(1) 0$

$= 0$

Similarily $a_y \times a_y = 0$; $a_z \times a_z = 0$.

$a_x \times a_y = |a_x||a_y| \sin 90° \hat{a}_n$

$= (1)(1)(1) a_z$

$= a_z$

Similarily $a_y \times a_z = a_x$; $a_z \times a_x = a_y$

Eg $\bar{A} \times \bar{B} = |\bar{B} \times \bar{A}|$

$\bar{A} = 2a_x + 3a_y + 2a_z$

$\bar{B} = 3a_x - 4a_y - 2a_z$

$\bar{A} \times \bar{B} = ((2a_x) + 3a_y + a_z) \times (3a_x - 4a_y - 2a_z)$

$= -8a_z - 4a_y + 9a_z - 6a_x + 3a_y - 4a_x$

$= -10a_x - a_y + a_z$

$|\bar{B} \times \bar{A}| = |(3a_x - 4a_y - 2a_z) \times (2a_x + 3a_y + a_z)|$

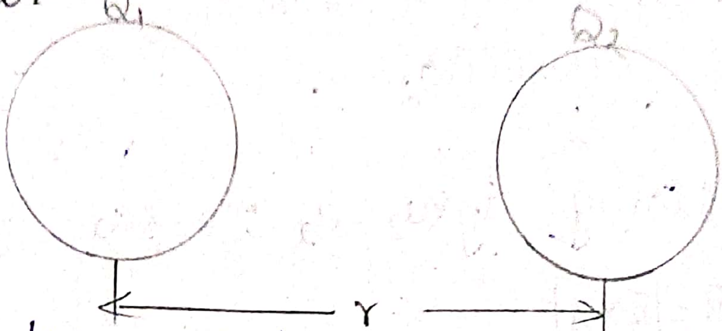
$= |9a_z + 3a_y - 8a_z - 4a_x - 4a_y - 6a_x|$

$= |-10a_x - a_y + a_z|$

$= 10a_x + a_y + a_z$

$\therefore \bar{A} \times \bar{B} = |\bar{B} \times \bar{A}|$

columbs law:-



* consider the two point chargers Q_1, Q_2 are seperated by distance r. The charge Q_1, Exists force on Q_2. While the charge Q_2 Exists a force on Q_1

The force exhausted in between them is repulsive force if the charges are same polarity which it attractive the charge are different polarities.

$\therefore$ The Force is directly proportional to the two charge are Q_1 & Q_2

$\therefore F \propto Q_1 \cdot Q_2$

The Force is inversely proportional to the square of distance between two charges

$F \propto \frac{1}{r^2}$

$F \propto \frac{Q_1 Q_2}{r^2}$

$F = \frac{K Q_1 Q_2}{r^2}$

$F = \frac{Q_1 Q_2}{4\pi \varepsilon r^2}$

$F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 r^2}$

$\because \mu = \mu_0 \mu_r$

$\mu_0 = 4\pi \times 10^{-7}$ H/m.

$k = \frac{1}{4\pi \varepsilon}$

$\varepsilon = \varepsilon_0 \varepsilon_r$

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

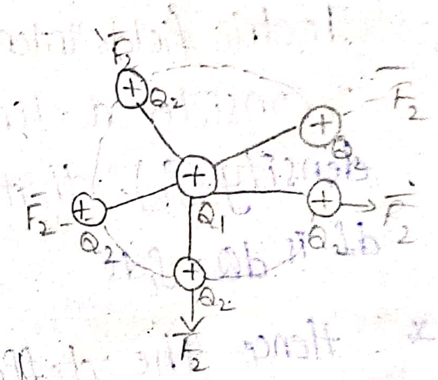
$\varepsilon_r = 1$ (air/free space)

Electric field intensity:-

Consider a point charge Q_1 & another charge Q_2
∵ Q_1 is the point charge, Q_2 is the moved charge
Thus there exists a region around a charge exerts a force on any other charge located in that region. is called Electric field of that charge:

* The Force Exerted per unit charge is called electric field intensity or Electric field strain.

* ∵ Force experienced by the charge Q_2 due to Q_1 is given by columbs law.



$$F_Q = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r R_{12}^2}\bar{a}_{12}$$

$$\frac{\bar{F_2}}{Q_2} = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_r R_{12}^2}\bar{a}_{12}$$

∵ Electric field intensity

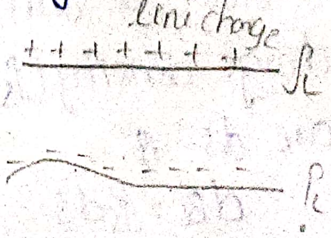
$$\bar{E} = \frac{F_2}{Q_2}$$

$$\therefore \bar{E} = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_r R_{12}^2}\bar{a}_{12} \; N/C = \frac{\text{Newton's}}{\text{Columbs}}$$

Line charge:-

The charge may be spreaded all along a line which may be finite or infinite Such charge uniformly distributed along a line is called Line charge

∴ The charge density of the line charge is denoted by ρ_L and defined as charge per unit length and is measured in columbs per meter



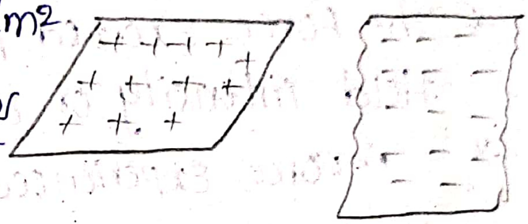
$$\therefore \rho_L = \frac{\text{total charge in columbs}}{\text{Total length in meters}}$$

$\therefore Q = \int_L dQ \qquad \because dQ = \rho_L dL$

Surface charge

If the charge is distributed uniformly over two dimensional surface then it is called a surface charge or sheet of charge. It is denoted as ρ_s & defined as the charge per unit surface area & it measure in coulumbs per meter^2 C/m^2.

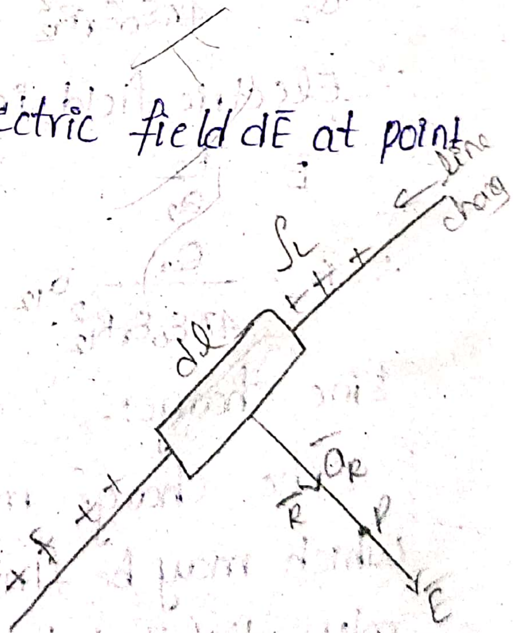
$$\therefore \rho_s = \frac{\text{Total charge in columbs}}{\text{Total area in } m^2}$$



Electric field intensity due to line charge.

Consider a line charge distribution having a charge density (ρ_L), that charge dQ on the differential length dl is $dQ = \rho_L dl$

* Hence the defferential electric field $d\bar{E}$ at point P due to dQ is given by



$$d\bar{E} = \frac{dQ}{4\pi\epsilon_0 R^2}\bar{a}_R$$

$$d\bar{E} = \frac{\rho_L dl}{4\pi\epsilon_0 R^2}\bar{a}_R$$

$$\int d\bar{E} = \int \frac{\rho_L dl}{4\pi\epsilon_0 R^2}\bar{a}_R$$

$$\bar{E} = \int_L \frac{\rho_L dl}{4\pi\epsilon_0 R^2}\bar{a}_R$$

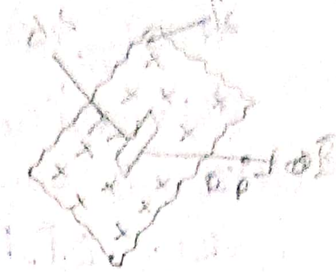
Electric field intensity due to surface charge

Consider a surface charge distribution having a charge density (ρ_s). the charge dQ on the differential are ds is

$$dQ = \rho_s ds$$

* Hence differential equation Electrical-field $d\bar{E}$ at a point p due to dQ is given by

$$d\bar{E} = \frac{dQ}{4\pi\epsilon_0 R^2}\bar{a}_R$$



$\therefore dQ = \rho_s ds$

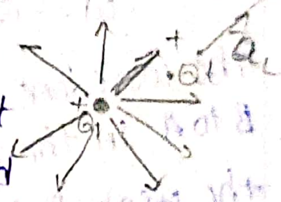
$$d\bar{E} = \oint_s \frac{\rho_s ds}{4\pi\epsilon_0 R^2}\bar{a}_R$$

Hence the total $\bar{E}$ at a point p is due to obtained by integrating E over the surface area on which charge is distribution $\therefore \bar{E} = \int_s \frac{\rho_s ds}{4\pi\epsilon_0 R^2}\bar{a}_R$.

22-09-22

Workdone in moving a point charge in Electric field

* consider a positive charge Q_1 & its Electric-field $\bar{E}$. if a positive test charge Q_t is placed in the ~~(speed)~~ field it will move due to ~~$\$$~~ force of repulsion



* Let the moment of charge Q_t is dl and denoted by unit vector $\bar{a}_L$
* i.e according coulumbs law the force exerted by the field $\bar{E}$ is given by $\bar{E} = \frac{\bar{F}}{Q_t}$

$$F = Q_t\bar{E} \text{ Newtons}$$

* The component of $\bar{F}$ in the direction unit vector $\bar{a}_L$ is given by $\bar{F}_L = \bar{F}\bar{a}_L$

$$F_L = Q_t\bar{E}\bar{a}_L$$

* i.e it is necessary to apply the force which is equal to & opposite to the force exerted by the field in the direction in the dl.

$$F_{applied} = -\bar{F}_L$$

$$F_{app} = -Q_t E a_L$$

* $\therefore$ differential workdone by an external force in moving the charge Q_t through a distance ~~dl~~ dl

against the dielectric field E is given by $W = F.dl$

differentiate

$dW = F_{app} \times dl$

$dW = -Q_t \bar{E} \bar{a}_L \times dl.$

Since $dl \times \bar{a}_L = \bar{dL}$ ⟶ using dot vector.

$dW = -Q_t E . \bar{dL}$

$\therefore \int dW = \int -Q_t E . \bar{dL}$

$$W = -\int_{initial}^{final} Q_t E . \bar{dL}$$

Potential difference.

The workdone in moving a point charge Q from point B to A in the Electricfield $\bar{E}$ is given by

$$W = -Q \int_B^A \bar{E}.\bar{dl}$$

The workdone per unit charge in a moving unit charge from B to A in the field $\bar{E}$ is called potential difference between the points B & A and it is denoted by B and it is measured in joules/coulumbs.

$$\text{potential difference} = \frac{\text{workdone}}{\text{charge}}$$

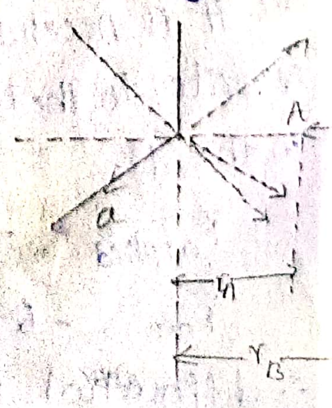
$$= \frac{-Q \int_B^A E.\bar{dl}}{Q}$$

$$= -\int_B^A \bar{E}.\bar{dl}$$

Potential due to point charge:

* consider a point charge located at the origin of a spherical coordinate the system, producing $\bar{E}$ radially in all directions.

* Assuming the free space, the field $\bar{E}$ due to a point charge Q at at point having radial distance r from origin is given by



$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2} \bar{a}_r$$

The differential line is in spherical system is

$\therefore \bar{dL} = dr \bar{a}_r + r d\theta \bar{a}_\theta + r \sin\theta d\phi \bar{a}_\phi$

$$V_{AB} = -\int_B^A \bar{E}\cdot d\bar{l}$$

$$V_{AB} = -\int_B^A \frac{Q}{4\pi\epsilon_0 r^2}\bar{a}_r\left(dr\,\bar{a}_r + r\,d\theta\,\bar{a}_\theta + r\sin\theta\,d\phi\,\bar{a}_\phi\right)$$

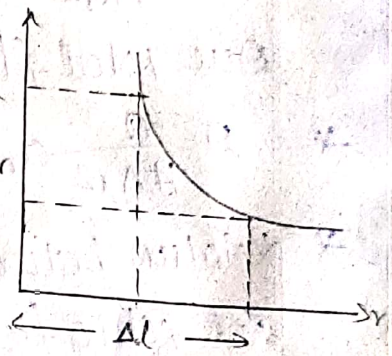
$$V_{AB} = -\int_B^A \frac{Q}{4\pi\epsilon_0 r^2}dr = \frac{-Q}{4\pi\epsilon_0}\int_B^A r^{-2}dr = \frac{-Q}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_B^A = \frac{+Q}{4\pi\epsilon_0}\left[\frac{1}{A}-\frac{1}{B}\right]$$

$$V_{AB} = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{r_A}-\frac{1}{r_B}\right] \text{ volts}$$

Potential gradient:-

The rate of change of potential with respect to distance is called potential gradient

$\therefore$ potential $V = -\int \bar{E}\cdot d\bar{l} = \frac{Q}{4\pi\epsilon_0 r}$



The potential decreases as distances of a point from the charge increases. From an Elementary length Δl is given by V_{AB}

$$V_{AB} = \Delta V = -E\cdot\Delta l$$

$\therefore$ Rate of change of potential difference

$$\frac{dv}{dl} = \lim_{\Delta l \to 0}\frac{\Delta V}{\Delta l} = \text{Potential gradient.}$$

The potential gradient is nothing but the slope of the graph of potential against distance at a point where Elementary length is considered.

24-04 Electric flux

The Total number of electric lines of force or flux in any particular Electric field is called Electric flux

properties of flux lines:-

* The flux lines starts from positive charge and terminate on the negative charge
* There are more number of lines i.e (thro) crowding of lines if Electric field is Stronger
* These lines are parallel and never cross each other
* The lines always enter or leave the charged Surface normally

electric flux density:-

The total Electric flux passing through the unit

unit Surface Area is called the Electric flux density

- It is denoted by $\bar{D}$ and is measured in columbs/m^2.
- $\therefore D = \frac{\psi}{S}$

* Where ψ = Total Electric flux in coulombs

S = Total Surface Area in meter^2 (m^2) $= 4\pi r^2$ of sphere.

$\therefore \bar{D} = \frac{Q}{4\pi r^2}$

Due Total flux = charge

* $\bar{D} = \frac{Q}{4\pi r^2}\bar{a}_r$ in vectors forms

* Relation between $\bar{D}$ & $\bar{E}$

$\therefore$ Electric field intensity $\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\bar{a}_r \longrightarrow (1)$

Electric flux density $\bar{D} = \frac{Q}{4\pi r^2}\bar{a}_r \longrightarrow (2)$

dividing (2) with (1)

$$\frac{(2)}{(1)} \quad \frac{\bar{D}}{\bar{E}} = \frac{\frac{\cancel{Q}}{\cancel{4\pi r^2}}\bar{a}_r}{\frac{\cancel{Q}}{4\pi\varepsilon_0 r^2}\bar{a}_r}$$

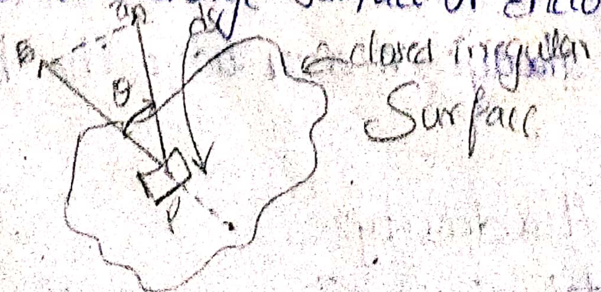
$$\frac{\bar{D}}{\bar{E}} = \frac{1}{\frac{1}{\varepsilon_0}}$$

$$\frac{\bar{D}}{\bar{E}} = \varepsilon_0$$

$$\bar{D} = \bar{E}\varepsilon_0$$

Gauss's law:-

The electric flux passing through any closed surface is equal to the total charge surface or enclosed by the surface



* The total surface enclosed by the irregular closed surface is Q in columbs.
* Consider a small defferential surface ds at a point P, the direction of $\bar{a}$ $\bar{D}$ as well as it's magnitude is going to change from the point to point on the surface
* i.e.: $\bar{ds} = a_n ds$ (∵ a_n = normal to the surface ds at point P.)
* we already know that.

$$D = \frac{\psi}{S}$$

$$d\psi = \bar{D}\,\bar{ds}$$

* Since $D = D_n$ = component of the $\bar{D}$ in the direction of noral surface ds.
* ∴ $d\psi = D_n \cdot ds$

$$D_n = \bar{D}\cos\theta$$

$$d\psi = \bar{D}(ds)\cos\theta$$

$$d\psi = \bar{D} \cdot \bar{ds}$$

$$\int d\psi = \oint_S \bar{D} \cdot \bar{ds}$$

$$\psi = Q$$

$$\therefore \psi = \int d\psi = \oint_S \bar{D} \cdot \bar{ds} = Q$$

28-09-22

* If there is a line charge with line charge density (ρ_L), Then $\psi = Q = \int_L \rho_L \, dl$
* If there is a surface charge with surface charge density (ρ_s) then $\psi = Q = \oint_S \rho_s \, ds$
* If there is a volume charge with volume charge density ρ_v then $\psi = Q = \int_V \rho_v \, dv$

Divergence in different co-ordinate system

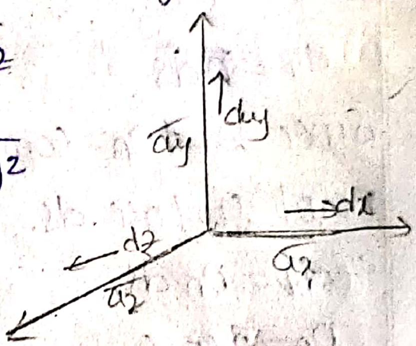
- There are classified into three types
  - i. cartesian / rectangular system
  - ii. cylindrical co-ordinate system
  - iii. spherical co-ordinate system

i. Cartesian / Rectangular system:

- consider a point (x, y, z) in rectangular co-ordinate system and $\bar{dL}$ is the differential length

i.e $\therefore \bar{dL} = dx\bar{a}_x + dy\bar{a}_y + dz\bar{a}_z$

$$|\bar{dL}| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$$



$$\Delta . \bar{A} = div\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

ii. cylindrical Co-ordinate system:-

- Consider a point (r, ϕ, z) in cylindrical co-ordinate system. $\bar{dL}$ is the differential length in r direction. $(r d\phi)$ is the differential length in ϕ direction. dz is the differential length in z direction

$$\bar{dL} = dr\bar{a}_r + r d\phi \bar{a}_\phi + dz\bar{a}_z$$

$$|\bar{dL}| = \sqrt{dr^2 + (r d\phi)^2 + (dz)^2}$$

$$\Delta . \bar{A} = div\bar{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

iii. Spherical co-ordinate system:

Consider a point (r, θ, ϕ) in spherical co-ordinate system. $\bar{dL}$ is the differential length

$$\therefore \bar{dL} = dr\bar{a}_r + r d\theta \bar{a}_\theta + r\sin\theta d\phi \bar{a}_\phi$$

$$|\bar{dL}| = \sqrt{(dr)^2 + (r d\theta)^2 + (r\sin\theta d\phi)^2}$$

$$\Delta . \bar{A} = div\bar{A} = \frac{1}{r^2}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

Maxwell's first equation:

According to guass's law

$$\psi = Q = \oint_S \bar{D} \cdot \bar{ds} \rightarrow (1)$$

Both side dividing per unit volume ΔV

$$\frac{Q}{\Delta V} = \frac{\oint_S \bar{D} \cdot \bar{ds}}{\Delta V}$$

Taking limits $\Delta V \rightarrow 0$

i.e $\lim_{\Delta V \to 0} \frac{Q}{\Delta V} = \lim_{\Delta V \to 0} \frac{\oint_S \bar{D} \cdot \bar{ds}}{\Delta V} \rightarrow (2)$

$\because$ The divergence of electric flux density $\bar{D}$ is given by

$$\Delta \cdot \bar{D} = \text{div}\bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$\because \text{div}\bar{D} = \lim_{\Delta V \to 0} \frac{\oint_S \bar{D} \cdot \bar{ds}}{\Delta V} \rightarrow (3)$ $\left(\because \lim_{\Delta V \to 0} \frac{Q}{\Delta V} = \rho \right)$

from (2) $\rho_v = \text{div}\bar{D}$

$$\boxed{\therefore \text{div}\bar{D} = \rho_v}$$

$$\boxed{\Delta \cdot \bar{D} = \rho_v}$$

This is the volume charge density around a point the above equation is called Maxwell's first equation / point form of guass law or guass law in differential form

Poissons & laplace equations:

We know that guass law in the point form

$$\Delta \cdot \bar{D} = \rho_v \rightarrow (1)$$

where ρ_v = volume charge density

$\bar{D}$ = Eletric flux density

$\because \bar{D} = \varepsilon \bar{E} \rightarrow (2)$

eq (2) in eq (1)

$\Delta \cdot \epsilon \bar{E} = \rho_v \rightarrow (3)$

$\because$ Ele potential gradient $\bar{E} = -\Delta V$

$\Delta \cdot \epsilon(-\Delta V) = \rho_v$

$\epsilon \Delta \cdot \Delta V = -\rho_v$

$$\boxed{\Delta^2 V = \frac{-\rho_v}{\epsilon}} \rightarrow (4)$$

The above equation is called poisson's equation. In certain region volume charge density is zero ($\Delta V = 0$), which is true for dielectric medium in the poissons equation then the

$$\Delta^2 V = 0$$

The above equation is called laplace equation, the Δ^2 operation is called the laplace of V

Δ^2 Operation in different co-ordinate system:-

* The potential(V) can be expressed in 3-cordinates systems $V(x, y, z)$, $V(r, \phi, z)$, $V(r, \theta, \phi)$ depending upon Δ^2 operation required for laplace equation must be used

* Cartesian cordinate system:-

$$\Delta V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z$$

$$\Delta \cdot \Delta V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$$

$$\Delta^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

* Cylindrical co-ordinate System:-

$$\Delta^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \left(\frac{\partial^2 V}{\partial z^2}\right) = 0$$

* Spherical co-ordinate System:-

$$\Delta^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

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pb:1 Determine wheather or not the potential fields Satisfy the laplace equations

(i) $V = x^2 - y^2 + z^2$ (ii) $V = r\cos\phi + z$ (iii) $r\cos\theta + \phi$

Sol: given $V = x^2 - y^2 + z^2$

$\Delta^2 V = ?$

$$\Delta V = \frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z}$$

$$= 2x - 2y + 2z$$

$$\Delta V = 2(x - y + z)$$

$$\Delta^2 V = 2\left[\frac{\partial}{\partial x}(x-y+z) + \frac{\partial}{\partial y}(x-y+z) + \frac{\partial}{\partial z}(x-y+z)\right]$$

$$= 2(1 \cancel{+1} + 1)$$

$$\Delta^2 V = 2 \neq 0$$

(ii) $V = r\cos\phi + z$

$$\Delta^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}(r\cos\phi + z)\right) + \frac{1}{r^2}\left(\frac{\partial^2 (r\cos\phi + z)}{\partial \phi^2}\right) + \frac{\partial^2}{\partial z^2}(r\cos\phi + z)$$

$$= \frac{1}{r}\frac{\partial}{\partial r}(r\cos\phi) + \frac{1}{r^2}\left(\frac{\partial}{\partial \phi}(r)(-\sin\phi)\right) + \frac{\partial}{\partial z}(1)$$

$$= \frac{1}{r}\cos\phi + \left(\frac{-1}{r^2}\right)r\cos\phi + 0$$

$$= \frac{1}{r}\left(\cos\phi - \cancel{\frac{1}{r}}\cos\phi\right)$$

$$= \frac{\cos\phi}{r}(1 - 1) = 0$$

(iii) given $V = r\cos\theta + \phi$

$$\Delta^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$$

$$= \frac{1}{r^2}\left(\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}(r\cos\theta + \phi)\right)\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}(r\cos\theta + \phi)\right)$$

$$+ \frac{1}{r^2\sin^2\theta}\left(\frac{\partial}{\partial \phi}\left(\frac{\partial}{\partial \phi}(r\cos\theta + \phi)\right)\right)$$

$$\Delta^2 u = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left[r^2\cos\theta\right]\right] + \frac{1}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}\sin\theta\left[r(-\sin\theta)\right]\right]$$

$$+ \frac{1}{r^2\sin^2\theta}\left[\frac{\partial}{\partial\phi}(1)\right]$$

$$\Delta^2 u = \frac{1}{r^2}\left(2r\cos\theta\right) + \frac{-1}{r^2\sin\theta}\left[\frac{\partial}{\partial\theta}\, r\sin^2\theta\right]$$

$$+ \frac{1}{r^2\sin^2\theta}(0)$$

$$\Delta^2 u = \frac{2}{r}\cos\theta - \frac{r}{r^2\cancel{\sin^2\theta}}\left(2\cancel{\sin\theta}\cos\theta\right) + 0$$

$$= \frac{2}{r}\cos\theta - \frac{1}{r\cancel{\sin\theta}}\, 2\cancel{\sin\theta}\cos\theta$$

$$= \frac{2}{r}\cos\theta - \frac{2\cos\theta}{r}$$

$$= \frac{2}{r}\cos\theta\left(1 - \frac{1}{1}\right)$$

$$= 0$$

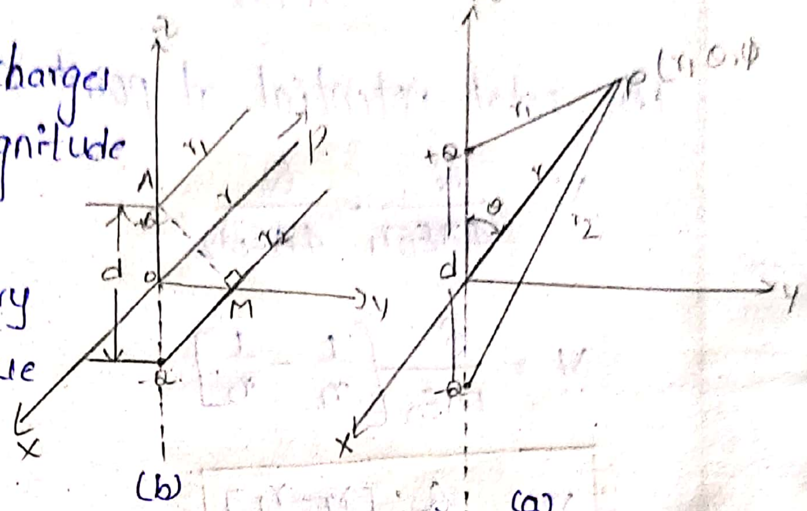
12/10/22

UNIT - II

ELECTRIC DIPOLE

Electric dipole:-

The Two point charges of a equal magnitude but opposite sign seperated by very small distance give a rise to an electric dipole.



consider an electric dipole the two point charges +Q, -Q. are seperated by very small distance 'd'.

consider a point $p(r, \theta, \phi)$ in spherical co-ordinate system

Let "O" be the mid point of "AB". The distance of point "p" from "A" is "r_1". while The distance of point "p" from "B" is "r_2". The distance of point "P" from point "O". is "r".

To find "E", we will find out potential "v" at point "P" due to an ~~dir~~ electric dipole. Then using

$$\bar{E} = -\Delta V \;/\; \bar{E} = -\nabla V$$

In spherical co-ordinates the potential at point "p" due to the charge "+Q" is given by

$$V_1 = \frac{+Q}{4\pi\varepsilon_0 r_1}$$

At potential at point "p" due to the charge "-Q" is given by

$$V_2 = \frac{-Q}{4\pi\varepsilon_0 r_2}$$

The Total potential at point "p" $V = V_1 + V_2$

$$V = \frac{Q}{4\pi\varepsilon_0 r_1} - \frac{Q}{4\pi\varepsilon_0 r_2}$$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{r_2 - r_1}{r_1 r_2}\right]}$$

The r_1, r_2, r are assume to be parallel to each other. AM is drawn perpendicular from "A" on "r_2"

$PB = BM + PM$ from fig(b)

$BM = PB - PM$ $PM = PA$

$BM = r_2 - r_1$

$(r_2 - r_1) = d\cos\theta$ $[\because BM = d\cos\theta]$

Since $r_1 = r_2 = r$

$$V = \frac{Q}{4\pi\varepsilon_0}\left[\frac{d\cos\theta}{r^2}\right]$$

$$\bar{E} = -\nabla V$$

$$\bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi\right]$$

$$= -\left[\frac{Qd\cos\theta}{4\pi\varepsilon_0}\frac{\partial}{\partial r}\left(\frac{1}{r^2}\right)\bar{a}_r + \frac{dQ}{r^3 4\pi\varepsilon_0}\frac{\partial}{\partial\theta}\cos\theta\,\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi}\right]$$

$$= -\left[\frac{Qd\cos\theta}{4\pi\varepsilon_0}\left[\frac{-2}{r^3}\right]\bar{a}_r + \frac{Qd}{4\pi\varepsilon_0 r^3}(-\sin\theta)\bar{a}_\theta\right]$$

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$$= + \frac{Qd\cos\theta}{2\pi\varepsilon_0 r^3}\bar{a}_r + \frac{Qd\sin\theta}{4\pi\varepsilon_0 r^3}\bar{a}_\theta$$

$$\boxed{\bar{E} = -\nabla V = \frac{Qd}{4\pi\varepsilon_0 r^3}\left[2\cos\theta\,\bar{a}_r + \sin\theta\,\bar{a}_\theta\right]}$$

13/11/24 **Dipole moment:-**

The product of charge and distance is called dipole moment and is denoted by "P" and is measured in columbs meters (C-m).

$$\therefore \bar{P} = Q\bar{d}$$

The dipole moment is measured in columbs meter

Now $\bar{P}\cdot\bar{a}_r = Q\bar{d}\cdot\bar{a}_r$

$$\bar{P}\cdot\bar{a}_r = Q(\bar{d}\cdot\bar{a}_r)$$

$$\bar{P}\cdot\bar{a}_r = Q|d||\bar{a}_r|\cos\theta$$

$$\bar{P}\cdot\bar{a}_r = Qd(1)\cos\theta$$

$$\bar{P}\cdot\bar{a}_r = Qd\cos\theta$$

Hence the expression of potential "V" can be expressed as

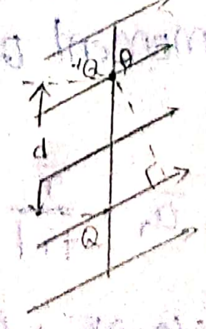
$$V = \frac{Qd\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\boxed{V = \frac{\bar{P}\cdot\bar{a}_r}{4\pi\varepsilon_0 r^2}}$$

$$\boxed{\bar{E} = \frac{Qd}{4\pi\varepsilon_0 r^3}\left[2\cos\theta\,\bar{a}_r + \sin\theta\,\bar{a}_\theta\right]}$$

Torque on electric dipole in an electric field:-

Consider an Electric dipole in an uniform Electric field $\bar{E}$. Making an angle θ w.r.t dipole axis



The charges "+Q" & "-Q" experienced a force due to an electric field $\bar{E}$ is equal to magnitude but opposite direction.

∴ Torque = Force × displacement

or

Torque = force × perpendicular distance of a separation.

$\tau = F \times l$ $\quad\quad \because E = \frac{F}{Q}$

$\tau = EQ \times l.$ $\quad\quad F = EQ.$

$\tau = EQl;$

Where L = perpendicular distance of separation between the two forces

$\sin\theta = \frac{l}{d}$

$l = d\sin\theta$

$\tau = EQ \times d \sin\theta$

While $\sin\theta$ is the cross product b/w the two the torque on the dipole is expressed as

$\tau = \bar{P} \times \bar{E}$

$\hookrightarrow$ cross

pb:1 A dipole having moment $\bar{P} = 3a_x - 5a_y + 10a_z$ nCm is locate at $Q(1,2,-4)$ in free space. Find V at $P(2,3,4)$

dipole moment of potential $V = \frac{\bar{P}\cdot\bar{a}_r}{4\pi\epsilon_0 r^2}$

$\bar{a}_r = ?$ $\bar{a}_r = \frac{\bar{r}}{|\bar{r}|}$

$\bar{r} = (2-1)\bar{a}_x + (3-2)\bar{a}_y + (4+4)\bar{a}_z$

$r = \bar{a}_x + \bar{a}_y + 8\bar{a}_z$

$|r| = \sqrt{1^2 + 1^2 + 8^2}$

$|r| = \sqrt{66} = 8.1$

$\bar{a}_r = \frac{r}{|r|} = \frac{\bar{a}_x + \bar{a}_y + 8\bar{a}_z}{\sqrt{66}}$

$\bar{P}.\bar{a}_r = (3a_x - 5a_y + 10a_z).\frac{(a_x + \bar{a}_y + 8a_z)}{\sqrt{66}}$

$= \frac{3 - 5 + 80}{\sqrt{66}}$

$= \frac{78}{\sqrt{66}} \times 10^{-9}$ cm

$= 9.60 \times 10^{-9}$ cm

$V = \frac{9.60 \times 10^{-9}}{4\pi \times 8.854 \times 10^{-12} (\sqrt{66})^2}$

$V = 1.30$ volts.

pb.2 Compute the torque for a dipole consisting of 1 micro columbs charges in the Electric field $\bar{E} = 10^3(2\bar{a}_x - \bar{a}_y - \bar{a}_z)$ V/m seperated by 1 mm & located on the z axis (only at the origin.

Given

$d = 1mm$

$Q = 1mc$

$d = 1 \times 10^{-3}$ m

$Q = 10^{-6}$ columbs

$\bar{E} = 10^3(2\bar{a}_x - \bar{a}_y - \bar{a}_z)$

$\tau = \bar{P} \times \bar{E}$

$\bar{P} = Qd$

$\bar{P} = 10^{-6} \times 10^{-3}$

$\bar{P} = 10^{-9}$

$\bar{P} = 1 \times 10^{-9}\, \bar{a}_z$ (due to z-axis)

$r' = 10^{-9} a_z \times (10^{3}(2\bar{a}_x - \bar{a}_y - \bar{a}_z)$

$= 10^{-9} a_z \times (10^{3} 2\bar{a}_x - 10^{3}\bar{a}_y - 10^{3}\bar{a}_z$

$x = \cancel{10^{-6} + 10^{-6}} \; (10^{-9} \times 10^{3}) + (10^{-9} \times 10^{3})$

$x =$

$= 10^{-6} 2\bar{a}_y - 10^{-6}\bar{a}_z$

$r = 10^{-6}(2-1)$

$= 2 10^{-6}\bar{a}_y - 10^{-6}\bar{a}_z$

15/10/22

Polarization:-

Consider an atom of a dielectric is consist of a nucleus with positive charge and negative charge in the form of revolving electrons in the orbits

The negative charge is thus consider to be in the form of cloud of electrons.

The number of positive charge is the same as negative charge and hence atom is electrically neutral

Both positive and negative charges can be assume to be point charges of equal amount conciding at the centre

Hence there cannot exist an electric dipole This is called unpolarized atom

The seperation between the nucleus and the centre of the electron cloud, Such an atom is called polarized atom.

The dipole gets aligned with the applied field this process is called polarization of dielectrics. There are two types of dielectric

i, Non-polar:-

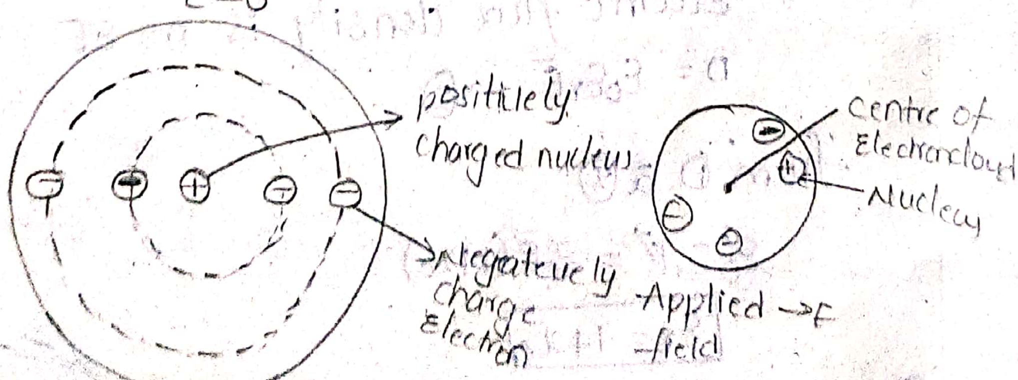
In non-polar molecules, the dipole arrangement is totally absent. In the absence of electric field E. it results only when an externally field $\bar{E}$ is applied to it

ii, polar:-

In polar molecules, the permanent displacements between the centres of positive and negative charges exists thus dipole arrangements exists without application of $\bar{E}$. The dipole experienced Torque and then align with the direction of the applied field $\bar{E}$. This is called polarization of polar molecules.

For example of Non-polar molecules are Hydrogen, Oxygen and rare gases.

The Example of polar molecules are water, Hydrochloric acid Sulphur dioxide.



unpolarized atom of a dielectric   polarized atom

Mathematical expression for polarization:

dipole moment $p = Qd$

The total dipole moment is to be obtained ...

Superposition principle is

$$P_{total} = Q_1\bar{d}_1 + Q_2 d_2 + Q_3 d_3 + \cdots Q_n d_n = \sum_{i=1}^{n\Delta V} Q_i d_i$$

$$P = \lim_{\Delta V \to 0} \frac{\sum_{i=1}^{n\Delta V} Q_i \bar{d_i}}{\Delta V}$$

The polarization $\bar{P}$ is defined as the total dipole moment per unit volume. it is measured in Coulumbs per square meters C/m^2

$$\bar{P} = \lim_{\Delta V \to 0} \frac{\sum_{i=1}^{n\Delta V} Q_i d_i}{\Delta V}$$

flux density in dielectric is $\bar{D} = \varepsilon_0 \bar{E} + \bar{P}$

for isotropic and non linear medium $\bar{P}$ & $\bar{E}$ are parallel to each other

$$\therefore \bar{P} = \chi_e \varepsilon_0 \bar{E}$$

Where χ_e = dimensionless quantity called Electric Susptability of the material

$$\bar{D} = \varepsilon_0 \bar{E} + \chi_e \varepsilon_0 \bar{E}$$

$$\bar{D} = \varepsilon_0 \bar{E} (\chi_e + 1) \longrightarrow ①$$

Since Electric flux density is $D = \varepsilon E$

$$D = \varepsilon_0 \varepsilon_r \bar{E} \longrightarrow ②$$

from ① & ②

$$\cancel{\varepsilon_0} \varepsilon_r \cancel{\vec{E}} = \cancel{\varepsilon_0} \cancel{\vec{E}} (1 + \chi_e)$$

$$\boxed{\varepsilon_r = 1 + \chi_e}$$

$(1+\chi_e)$ is defined as relative permitivity or the dielectric constant of the dielectric material

21-02-22

Boundary conditions:-

* When an electric field passes from one medium to another medium.
* The conditions existing at the boundary of the two media when field passes from one medium to other are called boundary conditions.

It is depending upon the nature of the media there are two boundary conditions

1. Boundary between conductor and free space (or) boundary between conductor and dielectric
2. Boundary between two dielectrics with different properties

* For studying the boundary conditions the maxwells eqns for electrostatics are required

$$\oint \bar{E} \cdot d\bar{l} = 0 \quad \& \quad \oint \bar{D} \cdot d\bar{s} = Q$$

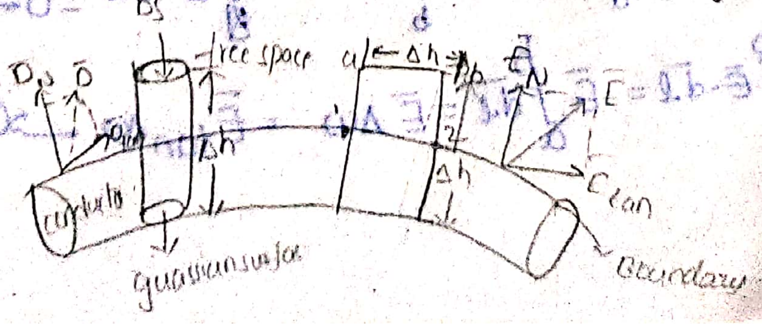
* The field intensity $\bar{E}$ is required to be decomposed into two components. They are

1. Tangentially to the boundary (E_{tan}) &
2. Normal to the boundary $(\bar{E}_N)$

$$\bar{E} = \bar{E}_{tan} + \bar{E}_N$$

Similarly decomposition is require for flux density $(\bar{D})$

Boundary condition between conductor and free space:-



Consider a boundary between conductor and free space the conductor is ideal having infinite conductivity Search conductors are copper, silver...

*] For ideal conductor it is known that

1. The field intensity intensity inside a conductor is zero and flux density inside a conductor is zero

2. No charge can exists with in a conductor the charge appears on the surface in the form of surface charge density

3 The charge density with in the conductor is zero

4. To determine the boundary conditions let us used closed path and gaussian surface

5. Let E Electric field intensity can be resolved into two components

i, The component Tangentially to the surface ($\bar{E}_{tan}$)

ii, The component normal to the surface ($\bar{E}_N$)

Consider a rectangular closed path abcda we know that enclosed surface

① $\oint \bar{E} \cdot d\bar{l} = 0$

$$\oint E \cdot d\bar{l} = \int_a^b \bar{E} \cdot d\bar{l} + \int_b^c \bar{E} \cdot d\bar{l} + \int_c^d \bar{E} \cdot d\bar{l} + \int_d^a \bar{E} \cdot d\bar{l} = 0 \quad (a)$$

$$\oint \bar{E} \cdot d\bar{l} = \int_a^b \bar{E} \cdot d\bar{l} + \int_b^c \bar{E} \cdot d\bar{l} + \int_d^a \bar{E} \cdot d\bar{l} = 0 \longrightarrow ①$$

$$\int_a^b \bar{E} \cdot d\bar{l} = E \int_a^b d\bar{l} = E \Delta w = \bar{E}_{tan} \Delta w \longrightarrow ②$$

$$\int_b^c \bar{E}\cdot\bar{dl} = \int_b^2 \bar{E}\cdot dl + \int_2^c \bar{E}\cdot\bar{dl} \longrightarrow$$

$$\int_b^c E\cdot dl = \bar{E}\,\frac{\Delta h}{2} = \bar{E}_N\frac{\Delta h}{2} \longrightarrow (3)$$

$$\int_d^a E\cdot\bar{dl} = -\bar{E}_N\,\frac{\Delta h}{2} \longrightarrow (4)$$

$$\bar{E}_{tan}\,\Delta w + \bar{E}_N\frac{\Delta h}{2} - \bar{E}_N\frac{\Delta h}{2} = 0$$

$$\bar{E}_{tan}\,\Delta w = 0$$

$$\therefore\ \bar{E}_{tan} = 0$$

$$\therefore\ \bar{D}_{tan} = 0$$

Tangentially component of electric field intensity and Electric flux density is zero at the boundary b/w conductor and free space

According to guass law $\oint \bar{D}\cdot\bar{ds} = Q$ To find the form of right circular cylinder

The surface integral must be calculated top & bottom and lateral

$$\int_{top} \bar{D}\cdot\bar{ds} + \int_{bottom} \bar{D}\cdot\bar{ds} + \int_{lateral} \bar{D}\cdot\bar{ds} = Q.$$

$$\int_{Top} \bar{D}\cdot\bar{ds} = Q.$$

$$\bar{D}\int_{top} ds = Q.$$

$$\bar{D}_N\,\Delta s = Q \longrightarrow (1)$$

∴ Surface charge intensity $Q = \rho_s \Delta s$ —— (2)

comparing Eq (1) & (2)

$$\bar{D}_N \Delta S = \rho_s \Delta s$$

$$\bar{D}_N = \rho_s$$

Normal component of the Electric flux density

$$\bar{D}_N = \epsilon \bar{E}_N$$

$$D_N = \epsilon_0 \bar{E}_N$$

$$\rho_s = \epsilon_0 \bar{E}_N$$

$$\bar{E}_N = \frac{\rho_s}{\epsilon_0}$$

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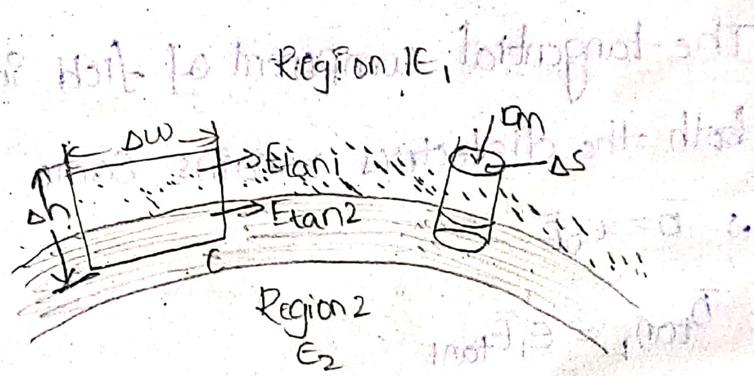
22-10

Boundary conditions b/w two dielectrics / two perfect dielectrics

Let us consider (two perfect dielectrics) the boundary between two perfect dielectric. one dielectric has ϵ_1 while the authour has permitivity ϵ_2

The $\bar{E}$ and $\bar{D}$ are to obtain again by resolving methods each into two components: tangential to the boundary and normal to the Surface

Consider a closed path abcd a rectangular in shape having elimantary hieght Δh & Eliminantary width Δw



$$\oint \bar{E}\cdot \bar{dL} = 0$$

$$\oint \bar{D}\cdot \bar{ds} = Q$$

$$\int_a^b \bar{E}\cdot\bar{dL} + \int_b^c \bar{E}\cdot\bar{dL} + \int_c^d \bar{E}\cdot\bar{dL} + \int_d^a \bar{E}\cdot\bar{dL} = 0$$

$\because \bar{E_1} = \bar{E_{1t}} + \bar{E_N}$

$\bar{E_2} = \bar{E_{2t}} + \bar{E_{2N}}$

$|E_{1t}| = E_{tan1}$; $|E_{2t}| = E_{tan2}$.

$|E_{1N}| = E_{1N}$; $|E_{2N}| = E_{2N}$

$\int_b^c$ and $\int_d^a$ becomes zero these are line integrals along Δh and $\Delta h \to 0$

$$\therefore \int_a^b \bar{E}\cdot d\bar{L} + \int_c^d \bar{E}\cdot d\bar{L} = 0$$

$$\bar{E}\int_a^b d\bar{L} + \bar{E}\int_c^d d\bar{L} = 0$$

$$\because \bar{E}\int_a^b d\bar{L} = \bar{E}_{tan_1}\Delta\omega$$

$$\bar{E}\int_a^b d\bar{L} + \bar{E}\int_c^d d\bar{L} = 0$$

$$\bar{E}\int_c^d d\bar{L} = -\bar{E}_{tan_2}$$

$$\therefore E_{tan_1}\Delta\omega + -\bar{E}_{tan_2}\Delta\omega = 0$$

$$\bar{E}_{tan_1} = +\bar{E}_{tan_2}$$

The tangential component of field intensity in the both the dielectrics remains same

$$\therefore \bar{D} = \epsilon_0 \bar{E}$$

$$\bar{D}_{tan_1} = \epsilon_1 \bar{E}_{tan_1}$$

$$\bar{E}_{tan_1} = \frac{\bar{D}_{tan_1}}{\epsilon_1}$$

$$\bar{D}_{tan_2} = \epsilon_2 \bar{E}_{tan_2}$$

$$\bar{E}_{tan_2} = \frac{\bar{D}_{tan_2}}{\epsilon_2}$$

$$\frac{\bar{D}_{tan_1}}{\bar{D}_{tan_2}} = \frac{\epsilon_1 \cancel{\bar{E}_{tan_1}}}{\epsilon_2 \cancel{\bar{E}_{tan_2}}}$$

$$\frac{\bar{D}_{tan_1}}{\bar{D}_{tan_2}} = \frac{\epsilon_1}{\epsilon_2} = \frac{\epsilon_{r_1}}{\epsilon_{r_2}}$$

The Tangential component of D undergoes some change across the interface Hence Tangential $\bar{D}$ is said to be discontinous across the boundary

To find Normal component

Let us use Guass law

$$\oint \bar{D}\cdot \bar{ds} = Q.$$

consider a guassian surface in the form of righ circular cylinder

$$\int_{Top} \bar{D}\cdot\bar{ds} + \int_{bottom} \bar{D}\cdot\bar{ds} + \int_{lateral} \bar{D}\cdot\bar{ds} = Q.$$

$\because \int_{lateral} \bar{D}\cdot ds = \Delta h = 0$

$$\therefore \int_{Top} \bar{D}\cdot\bar{ds} + \int_{bottom} \bar{D}\cdot\bar{ds} = Q. \rightarrow (1)$$

$$\int_{TOP} \bar{D}\cdot\bar{ds} = \bar{D}\int_{TOP} \bar{ds} = \bar{D}_{N_1}\Delta S$$

$$\int_{bottom} \bar{D}\cdot ds = \bar{D}\int_{bottom} \bar{ds} = -\bar{D}_{N_2}\Delta S$$

from (1)

$$\bar{D}_{N_1}\Delta S - \bar{D}_{N_2}\Delta S = Q$$

$$\because \rho_s \Delta S = Q$$

$$\bar{D}_{N_1}\Delta S - \bar{D}_{N_2}\Delta S = \rho_s \Delta S$$

$$(\bar{D}_{N_1} - \bar{D}_{N_2})\Delta S = \rho_s \Delta S$$

$$\rho_s = \bar{D}_{N_1} - \bar{D}_{N_2} \qquad \because \rho_s = 0$$

$$\bar{D}_{N_1} - \bar{D}_{N_2} = 0$$

$$\bar{D}_{N_1} = \bar{D}_{N_2}$$

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$\because \bar{D} = \epsilon \bar{E}$

$\bar{D}_{1N} = \epsilon_1 \bar{E}_{1N}$

$\bar{D}_{1N} = \epsilon_1 \bar{E}_{1N} \rightarrow (1)$

$\bar{E}_{1N} = \frac{\bar{D}_{1N}}{\epsilon_1} \rightarrow (1)$

$\bar{E}_{2N} = \frac{\bar{D}_{2N}}{\epsilon_2} \rightarrow (2)$

$\bar{D}_{2N} = \epsilon_2 \bar{E}_{2N} \rightarrow (2)$

$$\cancel{\frac{(1)}{(2)} = \frac{\bar{D}_{1N}}{\bar{D}_{2N}} = \frac{\epsilon_1 \bar{E}_{1N}}{\epsilon_2 \bar{E}_{2N}}}$$

$$\frac{(1)}{(2)} \quad \frac{\bar{E}_{1N}}{\bar{E}_{2N}} = \frac{\frac{\cancel{\bar{D}_{1N}}}{\epsilon_1}}{\frac{\cancel{\bar{D}_{2N}}}{\epsilon_2}}$$

$$\frac{\bar{E}_{1N}}{\bar{E}_{2N}} = \frac{\epsilon_2}{\epsilon_1}$$

$$\frac{\bar{E}_{1N}}{\bar{E}_{2N}} = \frac{\epsilon_2}{\epsilon_1} = \frac{\epsilon_{r_2}}{\epsilon_{r_1}}$$

The Normal components of diectric field intensity $\bar{E}$ are inversely proportional to the relative permitivities two media

Hence the Normal component of flux density $\bar{D}$ is continous at the boundary between the two perfect dielectrics

25-10-22 Concept of a capacitance:-

Consider two magnetic materials m_1, m_2 which are placed dielectric medium having permittivity ϵ

M_1 carries ~~to~~ positive charge Q. While m_2 carries negative charge equal in magnitudes as Q

dielectric field is normal to the conductor surface and the electric flux directed from m_1 towards m_2 in a system.

The potential difference between of m_1 & m_2

The ratio magnitude of the total charge on any one of the two conductors and potential difference between the conductors is called the capacitance of the two conductors. it denoted by C

$$C = \frac{Q}{V}$$

and is measured in faradays (or) columb/volts

Obtain form of guass law using $Q = \oint \bar{D} \cdot \bar{ds}$

$$\because \bar{D} = \epsilon_0 \bar{E}$$

$$\therefore Q = \oint \epsilon_0 \bar{E} \cdot d\bar{s}$$

$$Q = \oint \epsilon \bar{E} \cdot ds$$

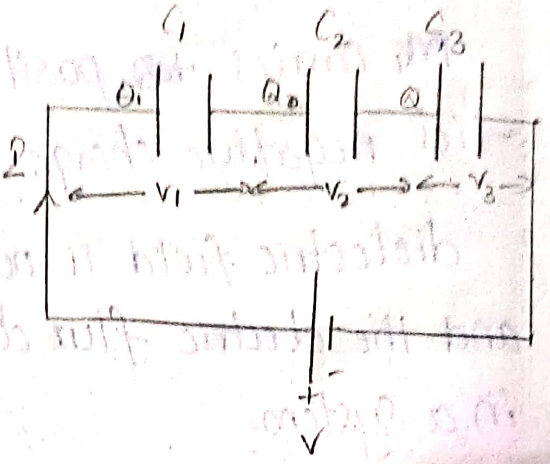
work done in moving positive charge to negative charge

$$V = -\int_L \bar{E} \cdot d\bar{L}$$

$$V = -\int_{-}^{+} \bar{E} \cdot d\bar{L}$$

$$C = \frac{\oint E \cdot ds}{-\int E \cdot dl}$$

Capacitancein series:-



$V = V_1 + V_2 + V_3$

$\because C = \frac{Q}{V}$

$Q = CV$

$Q_1 = Q = C_1 V_1 \Rightarrow V_1 = \frac{Q}{C_1}$

$Q_2 = Q = C_2 V_2 \Rightarrow V_2 = \frac{Q}{C_2}$

$Q_3 = Q = C_3 V_3 \Rightarrow V_3 = \frac{Q}{C_3}$

Let us consider a capacitors are connected in series. the acrossing the voltage V.

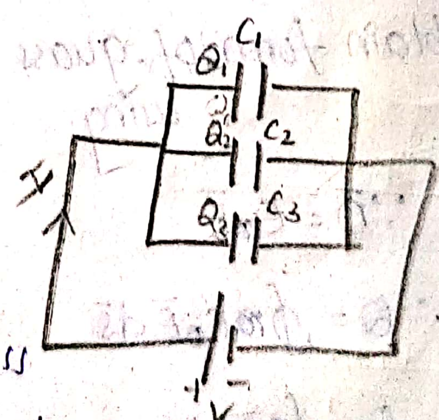
$V = V_1 + V_2 + V_3$

$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$

$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$

$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots$

Capacitance in parallel



Let us consider the capacitor are connected in parallel C_1, C_2, C_3 across the potential voltage V. In the parallel connection The total charge is Q and voltage is same V

The Total charge $Q = Q_1 + Q_2 + Q_3$

$\because Q = CV$

$Q_1 = C_1 V$

$Q_2 = C_2 V$

$Q_3 = C_3 V$

$Q = Q_1 + Q_2 + Q_3$

$Q = C_1 V + C_2 V + C_3 V$

$Q = (C_1 + C_2 + C_3) V$

$\frac{Q}{V} = C_1 + C_2 + C_3$

$C = C_1 + C_2 + C_3$

Energy stored in capacitor

Consider a parallel plate capacitor it is supplied with the voltage V.

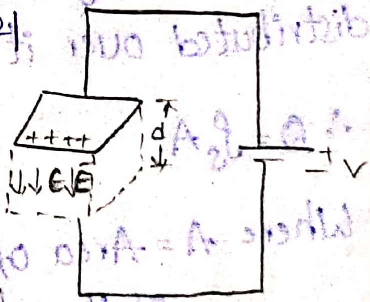
Let $\bar{a}_N$ is the direction of normal to the plates

$\therefore \bar{E} = \frac{V}{d} \bar{a}_N$

The Energy stored is given by $W_E = \frac{1}{2} \int_{VOL} \bar{D} \cdot \bar{E} \, dv$

$$W_E = \frac{1}{2} \int_{VOL} \epsilon \bar{E} \cdot \bar{E} \, dv \qquad \bar{E} \cdot \bar{E} = |E^2|$$

$$= \frac{1}{2} \int_{VOL} \epsilon \frac{V^2}{d^2} \, dv \qquad \bar{E} = \frac{V}{d}$$



$$= \frac{1}{2} \frac{\epsilon V^2}{d^2} \int_{VOL} dv$$

$$= \frac{1}{2} \frac{\epsilon V^2}{d^2} V \qquad \int_{VOL} dv = \text{volume} = A \times d$$

$$= \frac{1}{2} \frac{\epsilon V^2}{d^2} (A \times d)$$

$$= \frac{1}{2} \frac{\epsilon V^2 A}{d} \qquad C = \frac{\epsilon A}{d}$$

$$= \frac{1}{2} C V^2$$

Energy density

$$W_E = \frac{1}{2}\epsilon \int_{vol} |E^2| dv$$

$$W_E = \frac{1}{2}\epsilon |\bar{E}|^2 \quad J/m^3$$

$$\bar{D} = \epsilon |\bar{E}| \quad \because \bar{E} = \frac{\bar{D}}{\epsilon}$$

$$W_E = \frac{1}{2}\frac{|D|^2}{\epsilon} = \frac{1}{2}\bar{D}\bar{E} \quad J/m^3$$

$$W_E = \frac{1}{2}\bar{D}\cdot\frac{\bar{D}}{\epsilon}$$

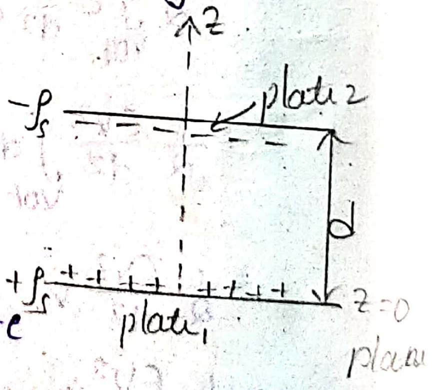
$$W_E = \frac{1}{2}\bar{D}\cdot\bar{E} \quad J/m^3$$

26-10-22

Parallel plate capacitor:-

It consists of 2 parallel metallic plates (capacitor) separated by distance d the space between the plates is filled with a dielectric of permitivity ϵ. The lower plate and upper plate carries with positive charge and negative charge respectively and is distributed over it with a charge density $+\rho_s$, $-\rho_s$

$\therefore Q = \rho_s A$



Where A = Area of Cross section of the plates in m^2

Assuming plate 1 to be infinite sheet of the charge

$$\bar{E}_1 = \frac{\rho_s}{2\epsilon}\bar{a}_N = \frac{\rho_s}{2\epsilon}\bar{a}_z \quad V/m$$

While for plate 2. $E_2 = \frac{-\rho_s}{2\epsilon}\bar{a}_N = \frac{-\rho_s}{2\epsilon}(-\bar{a}_z)$

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$\therefore \bar{E} = \bar{E}_1 + \bar{E}_2$

$$\bar{E} = \frac{\rho_s}{2\epsilon}\bar{a}_z - \frac{\rho_s}{2\epsilon}(-\bar{a}_z)$$

$$= \frac{\rho_s}{\epsilon}\bar{a}_z.$$

The potential difference is given by $V = -\int_{upper}^{lower} \bar{E}\cdot\bar{dL}$

$\because \bar{dL} = dx\bar{a}_x + dy\bar{a}_y + dz\bar{a}_z$

$$\therefore V = -\int_{z=d}^{z=0} \frac{\rho_s}{\epsilon}\bar{a}_z \cdot (dx\bar{a}_x + dy\bar{a}_y + dz\bar{a}_z)$$

$$= -\int_{d}^{0} \frac{\rho_s}{\epsilon} dz$$

$$= -\frac{\rho_s}{\epsilon}\int_{d}^{0} dz$$

$$= \frac{-\rho_s}{\epsilon} z\Big]_d^0$$

$$= \frac{-\rho_s}{\epsilon}[0-d]$$

$$= \frac{\rho_s d}{\epsilon}$$

The capacitance of the charge to voltage

$$Q = CV$$

$$C = \frac{Q}{V}$$

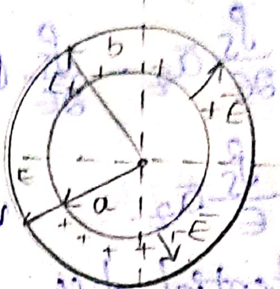
$$C = \frac{\rho_s A}{\frac{\rho_s d}{\epsilon}}$$

$$C = \frac{A\epsilon}{d}$$

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

Spherical capacitance:

Consider a spherical capacitor formed two concentric spherical conducting shells. of radius a & b



considering a guassian surface as a sphere of a radius r it can be obtained by $\bar{E}$ is in radial direction given by $E = \frac{Q}{4\pi\varepsilon r^2}\bar{a}_r \left(\frac{V}{m}\right)$

The potential difference (betw) is workdone in moving unit positive charge against the direction of $\bar{E}$.

i.e from b to a.

$$\therefore V = -\int \bar{E}\cdot\bar{dL} = -\int_b^a \bar{E}\cdot\bar{dL}$$

$$= -\int_b^a \frac{Q}{4\pi\varepsilon r^2}\bar{a}_r\cdot\bar{dL}$$

$$\because \bar{dL} = dr\,\bar{a}_r$$

$$V = -\int_b^a \frac{Q}{4\pi\varepsilon r^2}\bar{a}_r\cdot dr\,\bar{a}_r$$

$$V = -\frac{Q}{4\pi\varepsilon}\int_b^a \frac{1}{r^2}dr$$

$$V = \frac{-Q}{4\pi\varepsilon}\left.\frac{-1}{r}\right]_b^a$$

$$V = \frac{-Q}{4\pi\varepsilon}\left(\frac{-1}{r}\right)\Big]_b^a$$

$$V = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{a}-\frac{1}{b}\right)$$

$$V = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b}\right)$$

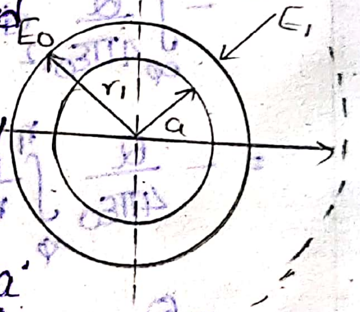
$\therefore Q = CV$

$$C = \frac{Q}{V}$$

$$C = \frac{Q}{\frac{Q}{4\pi\varepsilon}\left(\frac{1}{a} - \frac{1}{b}\right)}$$

$$C = \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)}$$

12-0-2 Isolated sphere coated with dielectric



Consider a single isolated sphere coated with a dielectric having permitivity ϵ_1, upto radius r_1.

The radius of inner sphere is 'a' it is placed in a free space. So outside the sphere ϵ_0. it carries a charge $+Q$.

$\because \epsilon = \epsilon_1$; $a < r < r_1$

$\epsilon = \epsilon_0$; $r > r_1$

$\therefore$ potential difference is workdone bringing unit positive charge from outer sphere $r = \infty$ to inner sphere $r = a$ against $\bar{E}$

$\therefore V = -\int \bar{E} \cdot \bar{dL}$

$$V = -\int_{\infty}^{a} \bar{E} \cdot \bar{dL}$$

$$V = -\int_{\infty}^{r_1} \bar{E} \cdot \bar{dL} - \int_{r_1}^{a} \bar{E} \cdot \bar{dL}$$

Now $a < r < r_1$

$$\bar{E}_1 = \frac{Q}{4\pi\epsilon_1 r^2}\bar{a}_r \rightarrow (1)$$

Now $r > r_1$

$$\bar{E}_2 = \frac{Q}{4\pi\epsilon_0 r^2}\bar{a}_r \rightarrow (2)$$

$\because \bar{dL} = dr\bar{a}_r$

$$V = -\int_{\infty}^{r_1}\frac{Q}{4\pi\epsilon_0 r^2}\bar{a}_r \cdot \bar{dL} - \int_{r_1}^{a}\frac{Q}{4\pi\epsilon_1 r^2}\bar{a}_r \cdot \bar{dL}$$

$$= -\int_{\infty}^{r_1}\frac{Q}{4\pi\epsilon_0 r^2}\bar{a}_r \cdot dr\bar{a}_r - \int_{r_1}^{a}\frac{Q}{4\pi\epsilon_1 r^2}\bar{a}_r \cdot dr\bar{a}_r$$

$$= -\int_{\infty}^{r_1}\frac{Q}{4\pi\epsilon_0 r^2}dr - \int^{a}\frac{Q}{4\pi\epsilon_1 r^2}dr$$

$$= -\frac{Q}{4\pi\epsilon_0}\int_{\infty}^{r_1}\frac{1}{r^2}dr - \frac{Q}{4\pi\epsilon_1}\int_{r_1}^{a}\frac{1}{r^2}dr$$

$$= \frac{-Q}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{r_1} - \frac{Q}{4\pi\epsilon_1}\left[\frac{-1}{r}\right]_{r_1}^{a}$$

$$= \frac{+Q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{\infty}\right] + \frac{Q}{4\pi\epsilon_1}\left[\frac{1}{a} - \frac{1}{r_1}\right]$$

$$= \frac{Q}{4\pi}\left[\frac{1}{\epsilon_0}\left(\frac{1}{r_1}\right) - 0 + \frac{1}{\epsilon_1}\left(\frac{r_1 - a}{ar_1}\right)\right]$$

$$= \frac{Q}{4\pi}\left[\frac{1}{\epsilon_0 r_1} + \frac{1}{\epsilon_1}\frac{(r_1 - a)}{ar_1}\right]$$

$$= \frac{Q}{4\pi r_1}\left[\frac{1}{\epsilon_0} + \frac{1}{\epsilon_1}\frac{(r_1 - a)}{a}\right]$$

$$\therefore C = \frac{Q}{V}$$

$$C = \frac{\cancel{Q}}{\frac{\cancel{Q}}{4\pi r_1}\left[\frac{1}{\epsilon_0} + \frac{1}{\epsilon_1}\left(\frac{r_1 - a}{a}\right)\right]}$$

$$C = \frac{4\pi r_1}{\frac{1}{\varepsilon_0} + \frac{1}{\varepsilon_1}\left(\frac{r_1 - a}{a}\right)}$$

$$\frac{1}{C} = \frac{\frac{1}{\varepsilon_0} + \frac{1}{\varepsilon_1}\left(\frac{r_1 - a}{a}\right)}{4\pi r_1}$$

$$\frac{1}{C} = \frac{1}{4\pi\varepsilon_0 r_1} + \frac{1}{\varepsilon_1 4\pi}\left(\frac{1}{a} - \frac{1}{r_1}\right)$$

it compare with $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{1}{C_1} = \frac{1}{4\pi\varepsilon_0 r_1}$$

$$C_1 = 4\pi\varepsilon_0 r_1$$

$$\frac{1}{C_2} = \frac{\left(\frac{1}{a} - \frac{1}{r_1}\right)}{4\pi\varepsilon_1}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

$$= \frac{1}{\frac{1}{4\pi\varepsilon_0 r_1} + \frac{\left(\frac{1}{a} - \frac{1}{r_1}\right)}{4\pi\varepsilon_1}}$$

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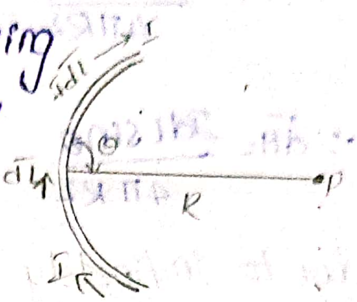
43 Magneto Statics, Amphere's law and force in Magnetic field

Biot Savarl law:-

Consider a conductor carrying direct current I, and a study magnetic field produced arou -nd it.



* The biot - Sawart law allows to obtain the differential magnetic field intensity $\overline{dH}$ produced at a point p. due to a differential current element $I\overline{dL}$

* It States that the proportional to the product of current(i) and differential length $\overline{dL}$.

* The Sine of the angle between the Element and the line joining point 'p' to the element and inversely proportional to the Square of the distance R between point "p" and the element

$$\therefore \overline{dH} \propto \frac{I dL \sin\theta}{R^2}$$

$$\overline{dH} = \frac{K I \overline{dL} \sin\theta}{R^2}$$

Where k = proportionality constant $= \frac{1}{4\pi}$

$$\overline{dH} = \frac{1}{4\pi} \frac{I \overline{dL} \sin\theta}{R^2}$$

$\overline{a}_R$ = unit vector in the direction from differential current element to point 'p then from cross productvector

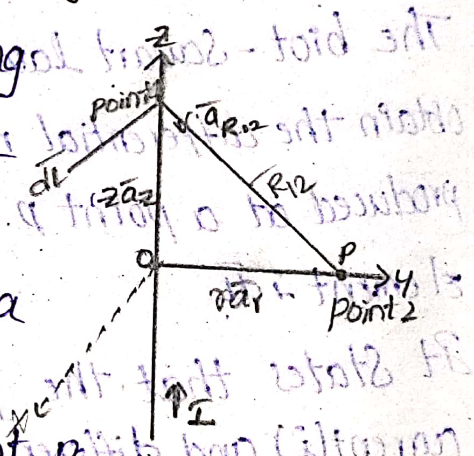
$$\bar{dL} \times \bar{a}_R = |dL||a_R| \sin\theta$$

$$= |dL| \sin\theta$$

$$\therefore \bar{dH} = \frac{I\, \bar{dL} \times \bar{a}_R}{4\pi R^2} \qquad \because a_R = \frac{R}{|R|}$$

$$\therefore \bar{dH} = \frac{I\, dL \sin\theta}{4\pi R^2}$$

$\bar{H}$ due to infinetely long straight conductors

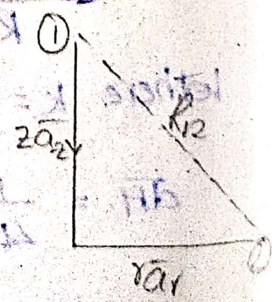


Consider an infinetely long straight conductor along z-axis. The current passing through the conductor is a direct current I amp. The field intensity $\bar{H}$ at a point p which is at a distance r from the z-axis. Small differential element at a point 1 along the z-axis at a distance z from the origin.

$$\therefore I\bar{dL} = I\, dz\, \bar{a}_z$$

The distance vector joining point 1 to point 2 is $\bar{R}_{12}$ & it can be written as $R_{12} = R\bar{a}_r - z\bar{a}_z$

$$\bar{a}_{R_{12}} = \frac{(\bar{R}_{12})}{|\bar{R}_{12}|} = \frac{r\bar{a}_r + z\bar{a}_z}{\sqrt{r^2+z^2}}$$



$$\bar{dL} \times \bar{a}_{R_{12}} = \begin{vmatrix} a_r & a_\phi & a_z \\ 0 & 0 & dz \\ r & 0 & -z \end{vmatrix}$$

Priyanka

$$d\bar{l} \times \bar{a}_{R_{12}} = -a_\phi(0 - r dz)$$

$$d\bar{l} \times \bar{a}_{R_{12}} = r dz\, a_\phi$$

$$\therefore d\bar{l} \times \bar{a}_{R_{12}} = \frac{r dz\, a_\phi}{\sqrt{r^2+z^2}}$$

According to biot savart's law $d\bar{H}$ at point 2 is

$$d\bar{H} = \frac{I d\bar{l} \times \bar{a}_{R_{12}}}{4\pi R_{12}^2} \quad (\text{amp-m})$$

$$\therefore d\bar{H} = \frac{I r dz\, a_\phi}{\sqrt{r^2+z^2}\, 4\pi R_{12}^2}$$

$$d\bar{H} = \frac{I r dz\, a_\phi}{4\pi \sqrt{r^2+z^2}\left(\sqrt{r^2+z^2}\right)^2}$$

$$d\bar{H} = \frac{I r dz\, a_\phi}{4\pi (r^2+z^2)^{3/2}} \rightarrow (1)$$

$\therefore$ put $z = r\tan\theta \Rightarrow z^2 = r^2\tan^2\theta$

$dz = r\sec^2\theta\, d\theta$ $\qquad$ θ limits :- $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

Integrating eq (1)

$$\bar{H} = \int_{-\pi/2}^{\pi/2} \frac{I r dz\, a_\phi}{4\pi (r^2+z^2)^{3/2}}$$

$$\bar{H} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{I r\, r \sec^2\theta\, d\theta\, a_\phi}{4\pi (r^2+z^2)^{3/2}}$$

$$\bar{H} = a_\phi \int_{-\pi/2}^{\pi/2} \frac{I r^2 \sec^2\theta\, d\theta}{4\pi (r^2 + r^2\tan^2\theta)^{3/2}}$$

$$\bar{H} = a_\phi \int_{-\pi/2}^{\pi/2} \frac{I r^2 \sec^2\theta \, d\theta}{4\pi r^3 (1+\tan^2\theta)^{3/2}}$$

$$\bar{H} = a_\phi \int_{-\pi/2}^{\pi/2} \frac{I \sec^2\theta \, d\theta}{4\pi r \sec^3\theta}$$

$$\bar{H} = a_\phi \int_{-\pi/2}^{\pi/2} \frac{I}{4\pi r} \cos\theta \, d\theta$$

$$\bar{H} = a_\phi \frac{I}{4\pi r} \sin\theta \Big]_{-\pi/2}^{\pi/2}$$

$$\bar{H} = a_\phi \frac{I}{4\pi r} \left[\sin \pi/2 - \sin(-\pi/2)\right]$$

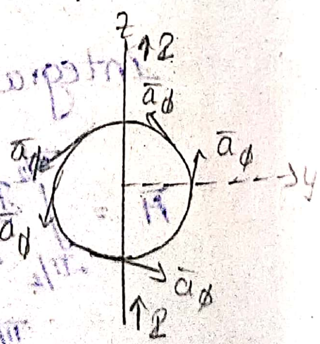
$$\bar{H} = a_\phi \frac{I}{4\pi r} (1-(-1))$$

$$\bar{H} = \frac{a_\phi I}{2\pi r}$$

$$\bar{H} = \frac{I}{2\pi r} a_\phi$$

Amperes circuit law:-

In magnetostatics, the complex problems can be solved using law called ampheres circuit law (or) Amperes work law



it states that the line integral of magnetic field intensity $\bar{H}$ around along a closed path is Exactly equal to the direct current enclosed by that path

$$\therefore \oint \bar{H} \cdot d\bar{l} = I$$

Proof of Ampere circuit law:

Consider a long straight conductor carrying direct current I placed along z axis Radius 'r', The point p is at a perpendicular distance r from the conductor consider $\bar{dL}$ at a point p which is in $\bar{a}_\phi$ direction, Tangential to circular path at point p.

$\therefore \bar{dL} = r d\phi \bar{a}_\phi$

$\therefore$ Biot Savart law due to infinitely long conductor

$$\bar{H} = \frac{I}{2\pi r} \bar{a}_\phi$$

$$\bar{H} \cdot \bar{dL} = \frac{I}{2\pi r} a_\phi \cdot r d\phi \bar{a}_\phi = \frac{I}{2\pi r} r d\phi \qquad \because \bar{a}_\phi \cdot a_\phi = 1$$

$$= \frac{I}{2\pi} d\phi$$

$$\oint \bar{H} \cdot \bar{dL} = \int_0^{2\pi} \frac{I}{2\pi} d\phi$$

$$= \frac{I}{2\pi} [\phi]_0^{2\pi}$$

$$= \frac{I}{2\pi}(2\pi - 0)$$

$$\oint \bar{H} \cdot \bar{dL} = I$$

Applications of Amperes work law:

(i) H due to infinetely long straight conductor.

$\therefore \bar{H} = H_\phi \bar{a}_\phi$

$\bar{dL} = r d\phi a_\phi$

$\bar{H} \cdot \bar{dL} = H_\phi a_\phi \cdot r d\phi a_\phi$

$\bar{H} \cdot \bar{dL} = H_\phi r d\phi$

According to ampere work law $\oint \bar{H} \cdot \bar{dL} = I$

$$\int_0^{2\pi} H_\phi r\, d\phi = I$$

$$H_\phi r [\phi]_0^{2\pi} = I$$

$$H_\phi r (2\pi - 0) = I$$

$$H_\phi = \frac{I}{2\pi r}$$

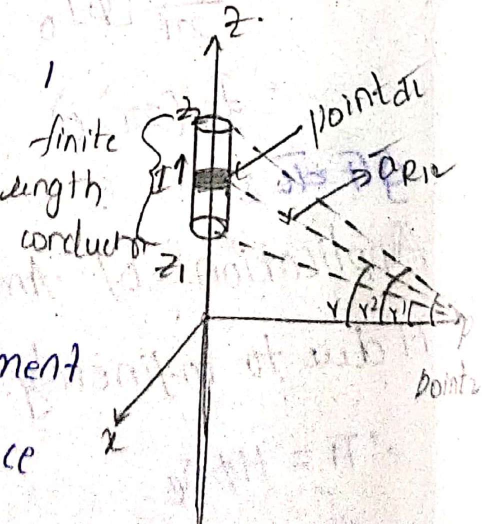
Hence H_ϕ at a point p is given by

$$\bar{H} = H_\phi \bar{a}_\phi$$

$$\bar{H} = \frac{I}{2\pi R} \bar{a}_\phi \ A/m$$

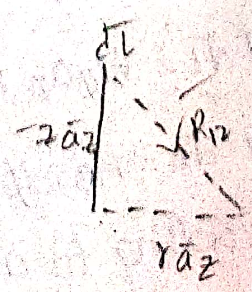
Magnetic field intensity due to straight conductor of finite length

Consider a conductor of finite length placed along z-axis. it carries a direct current "i", the $\perp^r$ distance of point p from z-axis is "r" the conductor is placed such that 1 and is at $z = z_1$ while the other is at $z = z_2$



Consider a differential element $d\bar{l}$ along z-axis at a distance z from origen therefore

$$d\bar{l} = dz\, \bar{a}_z$$



$$\bar{a}_{R_{12}} = \frac{R_{12}}{|R_{12}|}$$

$$\bar{R}_{12} = r a_R - z a_z$$

So $= \frac{r a_R - z \bar{a}_z}{\sqrt{r^2 - z^2}}$

$d\bar{l} \times \bar{a}_{R_{12}} = r dz a_\phi$ by det

$$\begin{vmatrix} r & \phi & z \\ 0 & 0 & dz \\ r & 0 & -z \end{vmatrix}$$

$a_\phi (0 - r dz)$

$I d\bar{L} \times \bar{a}_{R12} = \frac{I r dz a_\phi}{\sqrt{r^2+z^2}}$

According to biot savarts law

$$dH = \frac{I d\bar{L} \times \bar{a}_{R_{12}}}{4\pi R_{12}^2}$$

$$dH = \frac{I r dz a_\phi}{4\pi \sqrt{r^2+z^2} \left(\sqrt{r^2+z^2}\right)^2}$$

$$= \frac{I r dz a_\phi}{4\pi (r^2+z^2)^{3/2}}$$

put $z = r\tan\alpha$ with limits α_1 to α_2

$z^2 = r^2 \tan^2\alpha$

$dz = r\sec^2\alpha \, d\alpha$

$z = z_1 = r\tan\alpha_1$

$z = z_2 = r\tan\alpha_2$

Applying integration for above eq

$$\int dH = \bar{H} = \int_{\alpha_1}^{\alpha_2} \frac{I r \, r \sec^2\alpha \, d\alpha \, a_\phi}{4\pi (r^2 + r^2 \tan^2\alpha)^{3/2}}$$

$$= \int_{\alpha_1}^{\alpha_2} \frac{I r^2 \sec^2\alpha \, d\alpha \, a_\phi}{4\pi (r^2)^{3/2} (1+\tan^2\alpha)^{3/2}}$$

$$= \int_{\alpha_1}^{\alpha_2} \frac{I \sec^2\alpha \, d\alpha \, a_\phi}{4\pi r (\sec^2\alpha)^{3/2}}$$

$$= \int_{\alpha_1}^{\alpha_2} \frac{I \, d\alpha \, a_\phi}{4\pi r \sec\alpha}$$

$$= \frac{I}{4\pi r} a_\phi \int_{\alpha_1}^{\alpha_2} \cos\alpha \, d\alpha$$

$$= \frac{I}{4\pi r} a_\phi (\sin\alpha)_{\alpha_1}^{\alpha_2}$$

$$\bar{H} = \frac{I}{4\pi r} (\sin\alpha_2 - \sin\alpha_1) a_\phi$$

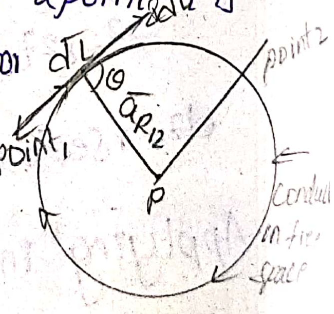
formula for magnetic flux density

$$B = \mu \bar{H} = \frac{\mu I}{4\pi r} (\sin\alpha_2 - \sin\alpha_1) a_\phi \left(\frac{wb}{m^2}\right)$$

Magnetic field intensity at the centre of circular conductor

Consider a current carrying conductor arranged in a circular form, the $\bar{H}$ at the centre of the circular loop is to be obtained

consider a differential length $\bar{dL}$ at a point, $I\bar{dL}$ is tangential to the circular conductor $\bar{dL}$ at point 1 where



θ = angle b/w $I.\bar{dL}$ & $\bar{a}_{R_{12}}$

∴ Cross product $I\bar{dL} \times \bar{a}_{R_{12}} = I|\bar{dL}| |a_{R_{12}}| \sin\theta$

$$= I dL \sin\theta \, \bar{a}_N$$

According to biot savarts law

$$\bar{dH} = \frac{I\bar{dL} \times \bar{a}_{R_{12}}}{4\pi R_{12}^2} = \frac{I dL \sin\theta \, \bar{a}_N}{4\pi R^2}$$

$$\oint \bar{dH} = \bar{H} = \frac{I \sin\theta}{4\pi R^2} \bar{a}_N \oint \bar{dL}$$

wher $\oint \bar{dL} = 2\pi R$ is the circumference of the circle

Replacing dl with $2\pi R$

$$= \frac{I\sin\theta}{4\pi R^2}\,\bar{a}_N\ 2\pi R$$

$$\bar{H} = \frac{I\sin\theta}{2R}\,\bar{a}_N$$

As Idl is the tangential to the circle R_{12} is Radius, Angle θ, $\angle\theta$ must be 90°

$$\bar{H} = \frac{I\sin 90^\circ}{2R}\,\bar{a}_N$$

$$\bar{H} = \frac{I}{2R}\,\bar{a}_N \quad \left(\frac{A}{m}\right)$$

Magnetic flux dencity

$$B = \mu \bar{H} = \frac{\mu_0 I}{2R}\,\bar{a}_N \ \ wb/m^2$$

Magnetic field intensity due to infinit sheet of current:-

* consider an infinite sheet of current in the $z=0$ plane, the Surface current density is $\bar{K}$. the current is flowing in y-direction, Hence $\bar{K} = K_y\bar{a}_y$
* Consider a closed path 1,2,3,4 the width of the path is 'b' the height is 'a'
* It is perpendicular to the direction of currents Hence in xz plane
* The current flowing across the distance 'b' is given by

  $\therefore I_{enclosed} = K_y b$
* consider the Magnetic lines of holes due to the current in $\bar{a}_y$ direction according to right hand thumb rule
* As current is flowing in y-direction. it canot have component in y-direction. So $\bar{H}$ as only

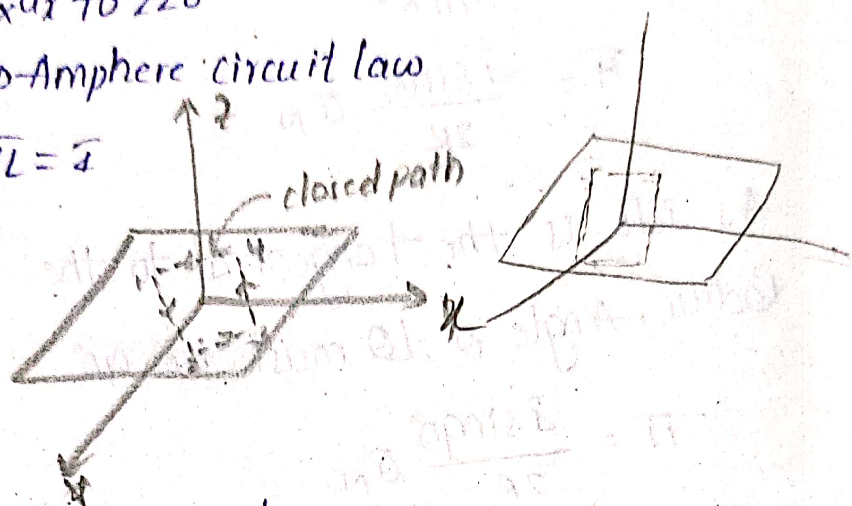
component in x-direction

$\bar{H} = H_x \bar{a}_x$ for $z>0$

$\bar{H} = -H_x \bar{a}_x$ for $z<0$

According to Amphere circuit law

$\oint \bar{H} \cdot d\bar{L} = I$



Evaluate the integral along the path 1-2-3-4-1

for path 1-2 $d\bar{L} = d_z \bar{a}_z$ since 3-4 is same. $\bar{H}$ is in x direction by while $\bar{a}_x \cdot \bar{a}_z = 0$. Hence 1-2 & 3-4 the integral

$\oint \bar{H} \cdot d\bar{L} = 0$

consider path 2-3 along which $d\bar{L} = dx \bar{a}_x$

$$\int_2^3 \bar{H} \cdot d\bar{L} = \int_2^3 (-H_x a_x)(dx \bar{a}_x)$$

$$= -H_x \int_2^3 dx = H_x b$$

$\because$ 1-4 are 4-1

$$\int_4^1 \bar{H} \cdot d\bar{L} = bH_x \quad \therefore \oint \bar{H} \cdot d\bar{L} = bH_x + bH_x = 2bH_x$$

$\therefore I_{enc} = k_y b$

$2bH_x = k_y b$

$H_x = \frac{k_y b}{2b} \quad H_x = \frac{1}{2} k_y$

An infinite sheet of current density $\bar{K}$ (A/m)

$\bar{H} = \frac{1}{2} \bar{K} \times \bar{a}_N$

Where $\bar{a}_N$ = unit vector normal form from the current sheet to the point at which $\bar{H}$ is to be obtained.

Force on a moving point charge. (Lorentz force eqn)

For a positive charge the force exerted on it is in the direction of E. This force is also called as Electric force

Now consider that a charge is placed in a steady magnetic field it experience a force only if it is moving then a magnetic force exerted on a charge Q moving with velocity $\bar{v}$ in a steady magnetic field $\bar{B}$ is give by

$$F_m = Q\bar{v} \times B$$

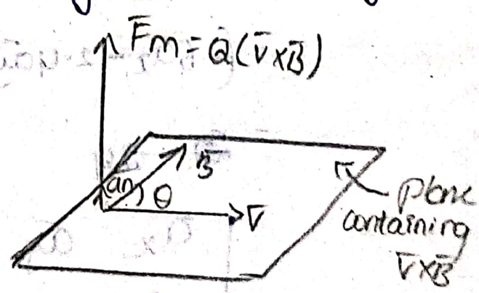
The Magnitude of the magnetic force is directly proportional to the Magnitude of $Q, \bar{v}$ & B and also sine of the angle between $\bar{v}$ & B

The direction of F_m is perpendicular to the plane containing $\bar{B}$ & $\bar{v}$

The Total force on a moving charge in the (process) presence of both Electric & magnetic field is given by

$$\bar{F_e} = Q\bar{E}$$

$$\bar{F_m} = Q(\bar{v} \times \bar{B})$$



$$\bar{F} = \bar{F_e} + \bar{F_m}$$

$$= Q\bar{E} + Q(\bar{v} \times \bar{B})$$

$$= Q(\bar{E} + (\bar{v} \times \bar{B})) \rightarrow ①$$

The eq① is also called as lorentz force equation which relates mechanical force to the Electrical force if the mass charge is 'm' then we can write

$$\bar{F} = ma$$

$$\bar{F} = m\frac{dv}{dt}$$

$$\bar{F} = Q[\bar{E} + (\bar{v} \times \bar{B})]$$

A point charge of Q

$Q = -1.2C$ has a velocity

$\bar{v} = 5\bar{a}_x + 2\bar{a}_y + 3\bar{a}_z$ m/s. Find the magnitude of the force, the charge if $\bar{E} = (-18\bar{a}_x + 5\bar{a}_y - 10\bar{a}_z)$ $\frac{V}{m}$ $\bar{B} = (-4\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z)$ Tesla, (c) Both are present in simultaniously

$F_e = Q\bar{E}$

$= (-1.2)(-18\bar{a}_x + 5\bar{a}_y - 10\bar{a}_z)$

$= 21.6\bar{a}_x - 6\bar{a}_y + 12\bar{a}_z$

$F_m = Q(\bar{v} \times \bar{B})$

$= (-1.2)[5\bar{a}_x + 2\bar{a}_y + 3\bar{a}_z] \times (-4\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z)]$

$= \cancel{-1.2[20-}$

$= (-6\bar{a}_x - 2.4\bar{a}_y + 3.6\bar{a}_z) \times (-4\bar{a}_x + 4\bar{a}_y + 3\bar{a}_z)$

$= \cancel{24 - 24}$

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ -6 & -2.4 & 3.6 \\ -4 & 4 & 3 \end{vmatrix}$$

$= \bar{a}_x(-7.2 - 14.4) - \bar{a}_y(-18 + 14.4) + \bar{a}_z(-24 - 9.6)$

$= \bar{a}_x(-21.6) - \bar{a}_y(-3.6) + \bar{a}_z(-33.6)$

Magnitude of electric force is given by $|F_e|$

$|F_e| = \sqrt{(21.6)^2 + (-6)^2 + (12)^2}$

$|F_e| = \sqrt{\quad + 36 + 144} = 25.42$

Magnitude of magnetic force is given by

$$|F_m| = \sqrt{(21.6)^2 + (3.6)^2 + (-33.6)^2}$$

$$|F_m| = 40.101\ N.$$

$\bar{F_\theta} = \bar{F_e} + \bar{F_m}$

$= 25.42$

$=$

$\bar{F} = \bar{F_e} + \bar{F_m}$

$\bar{F} = 21.6\bar{a_x} - 6\bar{a_y} + 12\bar{a_z} + -21.6\bar{a_x} - 3.6\bar{a_y} - 33.6\bar{a_z}$

$\bar{F} = -2.4\bar{a_y} - 21.6\bar{a_z}$

$|F| = \sqrt{(-2.4)^2 + (21.6)^2}$

$|F| = 21.70$

Force on a differential current element:

The Force Exerted on a differential Element of charge dQ moving in study magnetic field is given by $d\bar{F} = dQ\bar{v} \times \bar{B}$ (N) $\rightarrow$ ①

Since $\bar{J} = \rho_v \bar{v} \rightarrow$ ②

Current density

$d\bar{F} = dQ\bar{v} \times \bar{B}$

differential Element of charge $dQ = \rho_v dv \rightarrow$ ③

Sub ③ in ① we get

$dF = \rho_v dv \bar{v} \times \bar{B}$

$dF = \bar{J} \times \bar{B}\, dv$

The force exerted on Surface current density is given by $dF = \bar{K} \times \bar{B}\, ds$

Similarily the Exerted on differential current Element is given by $\because \bar{K}.ds = Id\bar{L}$

$$d\bar{F} = Id\bar{L} \times \bar{B}$$

integrate above eq over a closed path we get

$$\boxed{\bar{F} = \oint I d\bar{L} \times \bar{B}} \quad I \times B \oint d\bar{L}$$

Force on a straight & long current conductor:-

If a conductor is straight & long carrying current conductor and the field $\bar{B}$ is uniform along it. then integrating differential force $d\bar{F}$ represented in above equation we get simple expression for the poles as

$$\bar{F} = I\bar{L} \times \bar{B}$$

The Magnitude of the force is given by

$$F = ILB\sin\theta \qquad \because \bar{A} \times \bar{B} = AB\sin\theta$$

The Magnetic field exerts a magnetic force on the Electrons which constituet the current 'I'

Pb:1 A conductor 6m long, lies along z-direction with a current of 2Amp in $\bar{a}_z$ direction. Find the force experienced by conductor if $\bar{B} = 0.08\,\bar{a}_x$ Tesla.

$$\bar{F} = I d\bar{L} \times \bar{B}$$

$$\bar{F} = 2(6)\bar{a}_z \times 0.08\,\bar{a}_x$$

$$\bar{F} = 12\bar{a}_z \times 0.08\,\bar{a}_x$$

$F = 0.96\hat{a}_y$ (Newtons)

08-11-22

Force between different current Element:-

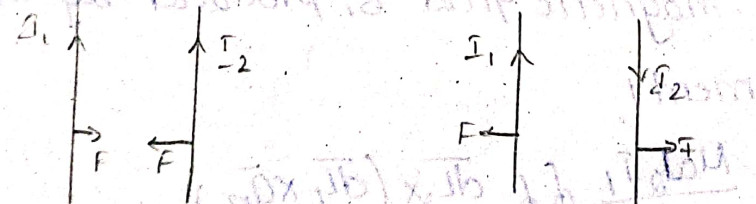
Consider that the two current carrying conductors are placed parallel to each other. Each of this conductor produces its own flux around it. When such two conductors placed closed to each other, there exists a force due to the interaction of two fluxes.

The force between the such parallel I carrying conductors depends on the direction of both two currents.

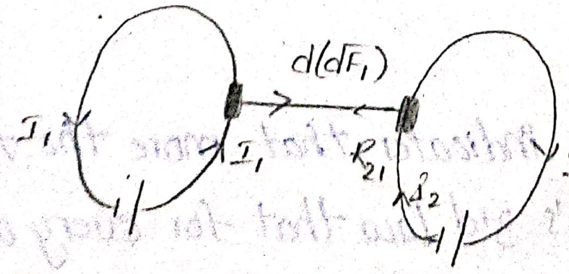
If the direction of both currents are same then the conductors experience a force of attraction.

If the direction of two currents are opposite to each other then the conductors experience a force of repulsion.

Consider two current elements $I_1 \bar{dL_1}$ & $I_2 \bar{dL_2}$



Force between two parallel current carrying conductor



Force between two current elements

The force Exerted on a differential Current element is given by

$$d(d\bar{F}_1) = I_1 d\bar{L}_1 \times d\bar{B}_2$$

$$d\bar{B}_2 = \mu_0 d\bar{H}_2 = \mu_0 \left[\frac{I_2 d\bar{L}_2 \times \bar{a}_{R_{21}}}{4\pi R_{21}^2}\right]$$

$$d(d\bar{F}_1) = I_1 d\bar{L}_1 \times \mu_0 \left[\frac{I_2 d\bar{L}_2 \times \bar{a}_{R_{21}}}{4\pi R_{21}^2}\right] \rightarrow (1)$$

eqn (1) integrating twice the total force $\bar{F}_1$ on current element 1 and due to current element 2 is given by

$$\bar{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1}\oint_{L_2} \frac{d\bar{L}_1 \times (d\bar{L}_2 \times \bar{a}_{R_{21}})}{R_{21}^2}$$

$$\bar{F}_2 = \frac{\mu_0 I_2 I_1}{4\pi} \oint_{L_1}\oint_{L_2} \frac{d\bar{L}_2 \times (d\bar{L}_1 \times \bar{a}_{R_{12}})}{R_{12}^2}$$

Similarily $\bar{F}_2$ Exerted on the current element to due to the magnetic field $\bar{B}_1$ produced by the current element 1.

$$\bar{F}_2 = \frac{\mu_0 I_2 I_1}{4\pi} \oint_{L_2}\oint_{L_1} \frac{d\bar{L}_2 \times (d\bar{L}_1 \times \bar{a}_{R_{12}})}{R_{12}^2}$$

$$\because \bar{F}_2 = -\bar{F}_1$$

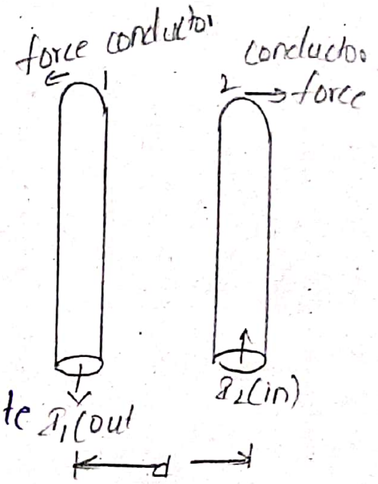
The Above condition indicates that more the forces $\bar{F}_1$ & $\bar{F}_2$ obey Newton's 3rd law that for every action there is Equal & opposite Reaction

Force between Two Straight, long Straight & parallel conductors carrying currents

consider two parallel long straight conductors of length l Each carrying current I_1 & I_2

let d be the distance of separation between the two conductors. the current in conductor 1 is moving out while that through conductor 2 is moving in. thus the two currents are in opposite direction

If the directions of the currents through the conductor attract & are same. then the two conductors attract eachother while. if the direction of currents through the conductors are opposite



The two conductor repel each other

(1) The force Exerted on a conductor is given by

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} \text{ Newtons}$$

28-11-22

UNIT - V

Self and Mutual inductance

Inductance:-

It is the property of the Material which opposes the rate of change of current to pass through it.

Self inductance:-

The property of a coil that oppose the change in current through it is called self inductance. it is denoted by the letter 'L' and is measured in Henry's'

- When a closed path or circuit carries a current (I), a magnetic field ($\bar{B}$) is produced. this causes a magnetic flux (ϕ) which is given by $\phi = \oint_s \bar{B} \cdot \bar{ds}$

- The flux linkage is defined as the product No. of turns and the total flux (ϕ) linkaging each of the turns. it is denoted by λ and is Measured in weber turn

[$\because B = \phi/A$, $\phi = BA$]

$$\therefore \lambda = N\phi \text{ (wb-turn)}$$

- The ratio of the total flux linkages to the current flowing through the circuit is called inductance and is given by

$$\boxed{L = \frac{N\phi}{I}} \text{ (web turn/Amp)} \rightarrow (1)$$

Method 2

- According to Self induced emf

$$e = L\frac{di}{dt}$$

$$\boxed{L = \frac{e}{\left(\frac{dI}{dt}\right)}} \rightarrow (2)$$

Method - II:

$$\phi = \frac{mmf}{reluctance}$$

$$\phi = \frac{NI}{\frac{l}{\mu_0 \mu_r a}}$$

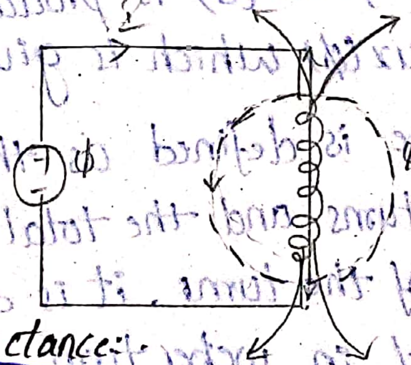
$$\phi = \frac{N I \mu_0 \mu_r a}{l} \longrightarrow (3)$$

Sub eq (3) in eq (1)

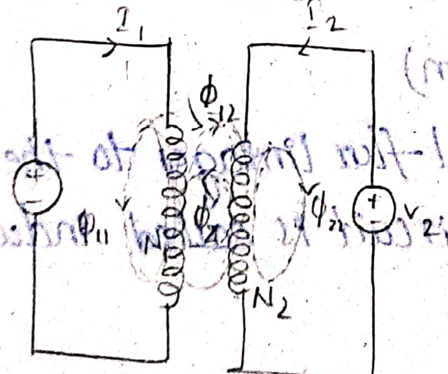
$$L = \frac{N}{\cancel{I}} \left(\frac{N \cancel{I} \mu_0 \mu_r a}{l} \right)$$

$$L = \frac{N^2 \mu_0 \mu_r a}{l}$$

$$\boxed{L = \frac{N^2}{S}}$$



Mutual inductance:



- The flux produced by circuit 1 due to current I_1 flowing through it is denoted by ϕ_{11} Ei
- Similarily the flux produced by circuit 2 due to current I_2 flowing through it is denoted by ϕ_{22}

1) The flux produced by links with the circuit itself and the other circuit. So the flux ϕ_{11} that links with the circuit 2 is denoted by ϕ_{12} and the flux ϕ_{22} that links with the circuit 1 is denoted by ϕ_{21}

2) The Mutual inductance b/w the two circuits is defined as the flux linkages of one circuit to the current in other circuit.

3) 1

$$M_{12} = \frac{N_2\phi_{12}}{I_1}$$

$$M_{21} = \frac{N_1\phi_{21}}{I_2} \quad (H)$$

2

$$e_m = M\frac{dI_1}{dt}$$

$$M = \frac{e_m}{\left(\frac{dI_1}{dt}\right)} \quad (H)$$

III

$$M = \frac{N_2\phi_{12}}{I_1}$$

$$\phi = \frac{mmf}{S} = \frac{N_1 I_1}{\frac{l}{\mu_0\mu_r a}}$$

$$= \frac{N_2}{I_1}\frac{N_1 I_1 \mu_0\mu_r a}{l}$$

$$= \frac{N_1N_2\mu_0\mu_r a}{l}$$

$$= \frac{N_1N_2}{S} \quad (H)$$

02-11-22

Coefficient of coupling b/w two circuits

2

When the two magnetic circuits kepts closed to each interacts with each magnetically through the flux linkages in the circuit due to the current in other circuit then the circuits are called Magnetically couple circuits.

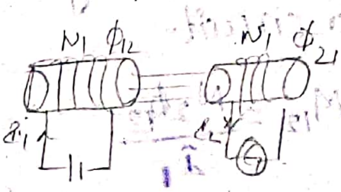
Self inductance of coil 1 $L_1 = \frac{N_1\phi_1}{I_1}$

self inductance of coil2 $= \frac{N_2\phi_2}{I_2} \rightarrow ②$

Mutual inductance coil1 $M_1 = \frac{N_2\phi_{12}}{I_1} \rightarrow ③$

Mutual inductance coil2 $M_2 = \frac{N_1\phi_{21}}{I_2} \rightarrow ④$

$\phi_{12} = k_1\phi_1 \quad \phi_{21} = k_2\phi_2$



$③ \Rightarrow M_1 = \frac{N_2k_1\phi_1}{I_1} \rightarrow ⑤$

$④ \Rightarrow M_2 = \frac{N_1k_2\phi_2}{I_2} \rightarrow ⑥$

eq⑤ x eq⑥

$$M_1 \times M_2 = \frac{N_2k_1\phi_1}{I_1} \times \frac{N_1k_2\phi_2}{I_2}$$

$$M \times M = \frac{N_1N_2k_1k_2\phi_1\phi_2}{I_1I_2}$$

$$M^2 = k_1k_2\left(\frac{N_1\phi_1}{I_1}\right)\left(\frac{N_2\phi_2}{I_2}\right) \qquad \therefore L_1 = \frac{N_1\phi_1}{I_1} \quad L_2 = \frac{N_2\phi_2}{I_2}$$

$$M^2 = k_1k_2L_1L_2 \qquad k_1 = k_2 = k$$

$$M^2 = k^2L_1L_2$$

$$k^2 = \frac{M^2}{L_1L_2}$$

$$k = \sqrt{\frac{M^2}{L_1L_2}}$$

$$\boxed{k = \frac{M}{\sqrt{L_1L_2}}}$$

* The two magnetic circuits are coupled together in series aiding and if m is mutual inductance between them then the effective inductance of the system is given by

$[L = L_1 + L_2 + 2M]$ Henry

Similarily two magnetic circuits L_1 & L_2 are magnitically coupled in series opposing then the effective inductance of the series system is given by $L_{eq} = L_1 + L_2 - 2M$

parallel Aiding $L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$ Mathri rayali

parallel opposing $L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$

06-12-22

pb: A Solenoid with $n_1 = 2000$, $r_1 = 2$ cm & $L_1 = 100$ cm is concentric with in a second coil of $n_2 = 4000$ $r_2 = 4$cm ; $L_2 = 100$cm. Find mutual inductance assuming free space inductance

Given $N_1 = 2000$ $\quad$ $N_2 = 4000$

$r_1 = 2$cm $\quad$ $r_2 = 4$ cm

$L_1 = 100$cm $\quad$ $L_2 = 100$ cm

$$M = \frac{N_1 N_2}{\frac{l}{\mu_0 \mu_r a}}$$

$$a = \frac{\pi d^2}{4}$$

$$M = \frac{N_1 N_2 \mu_0 \mu_r a}{l}$$

$$d = 2r$$

Magnetic field intensity $H_1 = \frac{N_1 I_1}{L_1}$

$$= \frac{2000 \times I_1}{100}$$

$$= 20 I_1 = 20 \times 10^{2} I_1 = 2000\, I_1 \text{ AT/m}$$

Magne field density $B = \frac{\phi}{A}$

$B = \mu H$

$B_1 = \mu H_1$

$B_1 = \mu_0 \mu_r H_1 = 4\pi \times 10^{-7} \times 1 \times 2000 I_1$

Total flux $\phi_1 = B_1 A$

$$\phi_1 = 2.512\times10^{-3} I_1 \times [\pi(2\times10^{-2})^2]$$

$$\phi_1 = 2.512\times10^{-3}\times\pi^2\times4\times10^{-4}$$

$$\phi_1 = 3.15\times10^{-6} I_1 \text{ weber}$$

$$M_2 = \frac{N_2\phi_{12}}{I_1} = \frac{400\times3.15\times10^{-6} I_1}{I_1} = 0.0126\,H$$

pb:- If a coil of 800 μH is magnetically couple to another coil of 200 μH. The co-efficient of coupling b/w two coils is 0.05. calculate inductance of two coils are connected (a) Series Aidind (b) Series opposing (c) parallel aidind (d) Parallel opposing

Given $L_1 = 800\times10^{-6}\,H$

$L_2 = 200\times10^{-6}\,H$

$k = 0.05$

(a) Series aidind

$$L_{eq} = L_1 + L_2 + 2M$$

Since $K = \frac{M}{\sqrt{L_1L_2}}$

$$M = k\sqrt{L_1L_2}$$

$$M = 2\times10^{-5}\,H$$

$$L_{eq} = 800\times10^{-6} + 200\times10^{-6} + 2\times2\times10^{-5}$$

$$= 1.04\times10^{-3}\,H$$

(b) Series opposing

$$L_{eq} = L_1 + L_2 - 2M$$

$L_{eq} =$ ~~[illegible]~~ $9.6\times10^{-4}\,H$

(c) parallel aiding

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$

$$= \frac{800\times10^{-6} \times 200\times10^{-6} - (2\times10^{-5})^2}{9.6\times10^{-4}}$$

$$= 1.6625 \times 10^{-12}\ H$$

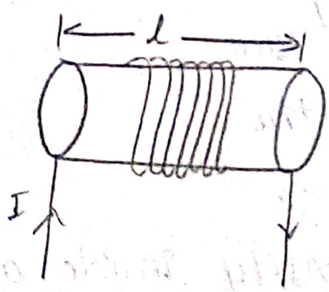
(d) parallel opposing

$$L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

$$= 1.596\times10^{-7}$$

$$= 1.53\times10^{-4}\ H$$

Inductance of a Solenoid:-



Consider a solenoid of (N) turns, let the current flowing through the solenoid be I. let the length of the solenoid 'l' & cross section Area be A (m^2)

*] The field intensity inside of the solenoid is given by $H = \frac{NI}{l}$ $\left(\frac{A\,turns}{m}\right)$

* Total flux linkages $= N\phi$ $\quad B = \frac{\phi}{A}$

$$= NBA$$

$$= N\mu HA \qquad B = \mu H$$

$$= \mu NA\left(\frac{NI}{l}\right)$$

Total flux linkages $= \frac{\mu N^2 A I}{l}$

The inductance of a Solenoid is given by

$$L = \frac{\text{Total flux linkags}}{\text{Total current}}$$

$$L = \frac{\frac{\mu N^2 A I}{l}}{I}$$

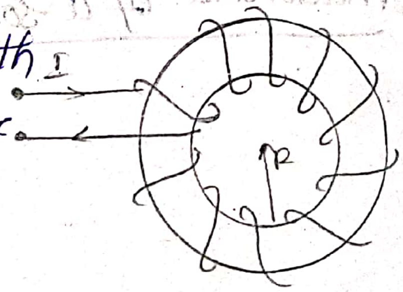
$$L = \frac{\mu N^2 A}{l} \text{ Henry}$$

$$L = \frac{N^2}{l/\mu_0 \mu_r A}$$

$$L = \frac{N^2}{S} \text{ Henry}$$

Torque <u>Inductance of Toroid</u>:-

Consider colloidal ring with turns N and carrying current I, Let the radius of the toroid be R



The magnetic flux density inside a toroidal ring is given by :-

$$B = \mu H$$

$$\phi = BA \quad \therefore B = \frac{\phi}{A}$$

$$B = \mu H$$

$$H = \frac{NI}{2\pi R}$$

$$\boxed{B = \frac{\mu N I}{2\pi R}}$$

Total flux linkages $= N\phi$

$$= NBA$$

$$= NA \frac{\mu N I}{2\pi R}$$

$$= \frac{\mu N^2 I A}{2\pi R}$$

Inductance of toraid is given by

$$L = \frac{\text{Total flux linkages}}{\text{Total current}}$$

$$L = \frac{\frac{\mu N^2 I A}{2\pi R}}{I}$$

$$L = \frac{\mu N^2 A}{2\pi R} \text{ Henrys.}$$

Where A = Area cross of toroid ring $= \pi r^2 \, (m^2)$

For a toroid with No of turns (N) & h = hight of the toroid with (r_1) has inner radius and (r_2) has outer radius, the inductance is given by $L = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$

$$L = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

pb: calculate the inductance of a solenoid of 200 turns wound tightly on a cylindrical tube of 6cm diameter the length of the tube is 60cm & the solenoid is in air.

Given that

Turns $N = 200$

diameter $d = 6cm = 6 \times 10^{-2} m$

length $l = 60 cm = 60 \times 10^{-2} m$

Solenoid inductance $L = \frac{\mu N^2 A}{l}$

$A = \pi r^2$

$r = \frac{d}{2} = \frac{6 \times 10^{-2}}{2} = 3 \times 10^{-2}$

$r^2 = (3 \times 10^{-2})^2$

$= 9 \times 10^{-4}$

$$L = \frac{\mu_0 \mu_r N^2 A}{l}$$

$$L = \frac{4\pi \times 10^{-7} \times (1) \times (200)^2 \times 9 \times 10^{-4} \times \pi}{60 \times 10^{-2}}$$

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$$L = \frac{4\pi^2 \times 10^{-7} \times 4 \times 10^{4} \times 9 \times 10^{-4}}{60 \times 10^{-2}}$$

$$L = \frac{\pi^2 \times 144 \times 10^{-7}}{6 \times 10^{-1}}$$

$$L = \frac{\pi^2 \times 144 \times 10^{-6}}{6}$$

$$L = \pi^2 \times 24 \times 10^{-6}$$

$$L = 2.368 \times 10^{-4} \text{ Henry}$$

Pb:2 A coil of 500 turns is wound on a closed iron ring of mean radius 10cm & cross sectional area of $3cm^2$ find the self inductance of the winding if the relative permibility of iron is 800

Give data

$N = 500$

radius $R = 10cm = 10 \times 10^{-2} m$

Area $A = 3cm^2 = 3 \times (10^{-2})^2 = 3 \times 10^{-4} m^2$

μ_r relative permibility $= 800$

$$L = \frac{\mu N^2 A}{2\pi R}$$

$$= \frac{\mu_0 \mu_r N^2 A}{2\pi R}$$

$$= \frac{4\pi \times 10^{-7} \times 8 \times 10^{2} \times 25 \times 10^{4} \times 3 \times 10^{-4}}{2\pi \times 10^{-1}}$$

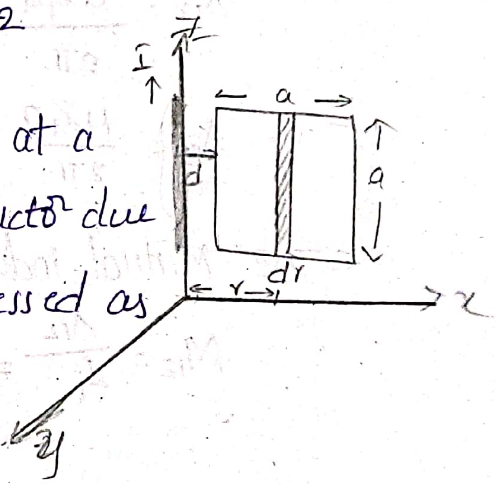
$$= 16 \times 25 \times 3 \times 10^{-4}$$

$$= 16 \times 75 \times 10^{-4}$$

Mutual inductance b/w a long straight wire & square loop lying in same plane

Consider a square loop with sides 'a', a straight long conductor is kept parallel to the longer side of the loop along Z-axis.

Consider a long straight wire is circuit 1 while a square loop is circuit - 2



The Magnetic field intensity at a distance of 'd' from long conductor due to current I_1, it can be expressed as using Amphere circuit law.

$$\oint \bar{H} \cdot \bar{dL} = I$$

$$H_1(2\pi r) = I_1$$

$$\boxed{H_1 = \frac{I_1}{2\pi r} \bar{a_\phi}} \longrightarrow (1)$$

Magnetic flux density

$$B_1 = \mu H_1$$

$$B_1 = \mu_0 \mu_r H_1$$

$$\boxed{B_1 = \mu_0 \mu_r \frac{I_1}{2\pi r} \bar{a_\phi}} \longrightarrow (2)$$

The flux linkages in circuit 2 due to current in circuit - 1

$$\lambda_{12} = \int_{S_2} B_1 \cdot dS_2$$

Since $dS_2 = a\,dr\,\bar{a_\phi}$

$$\lambda_{12} = \int_{d}^{d+a} \frac{\mu I_1}{2\pi r} \bar{a}_\phi \cdot (a\,dr\,\bar{a}_\phi)$$

$$\lambda_{12} = \frac{\mu I_1 a}{2\pi} \int_{d}^{d+a} \frac{1}{r} dr$$

$$\lambda_{12} = \frac{\mu I_1 a}{2\pi} \log r \Big]_{d}^{d+a}$$

$$\lambda_{12} = \frac{\mu I_1 a}{2\pi} \left(\log(d+a) - \log d \right)$$

$$\lambda_{12} = \frac{\mu I_1 a}{2\pi} \log_e \left(\frac{d+a}{d} \right)$$

$$\lambda_{12} = \frac{\mu I_1 a}{2\pi} \log_e \left(1 + \frac{a}{d} \right)$$

∴ The Mutual inductance

$$M_{12} = \frac{\lambda_{12}}{I_1} = \frac{\frac{\mu \cancel{I_1} a}{2\pi} \log_e\left(1+\frac{a}{d}\right)}{\cancel{I_1}}$$

$$= \frac{\mu a}{2\pi} \ln\left(1+\frac{a}{d}\right)$$

(or)

Mutual inductance b/w a long wire Straight wire & rectangular loop in same plane M_{12} =

$$M_{12} = \frac{\lambda_{12}}{I_1} = \frac{\mu b}{2\pi} \ln\left(1+\frac{a}{d}\right)$$

3. Energy stored in a magnetic field

Consider a differential value in Magnetic field B, Consider at the top & bottom surfaces of a differential value conducting sheets ΔI are present since energy stored magnetic field

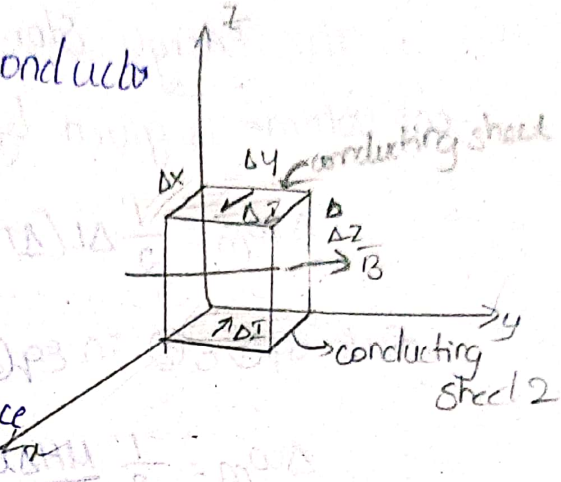
$$E = \frac{1}{2} L I^2$$

The inductance of the conductor

$L = \frac{N\phi}{I}$

$\because N = 1$

$L = \frac{\phi}{I}$



The differential inductance

$\Delta L = \frac{\Delta\phi}{\Delta I}$

The Magnetic flux

$\phi = BA$

$\Delta\phi = B\Delta S$

$\Delta L = \frac{B\Delta S}{\Delta I}$

where ΔS = differential Surface area $= \Delta x \Delta z$

$\therefore \Delta L = \frac{B\Delta x\Delta z}{\Delta I}$

$B = \mu H$

$$\boxed{\Delta L = \frac{\mu H \Delta x \Delta z}{\Delta I}} \rightarrow (1)$$

The differential current ΔI can be in terms of magnetic field intensity H. The current flowing through the conduction sheet present at the top & bottom is in y-direction.

$\oint H \cdot dL = I$

$H \cdot dL = \Delta I$

Since $\Delta L = \Delta y$

$$\boxed{\therefore \Delta I = H\Delta y} \rightarrow (2)$$

∴ The energy stored in a inductance of a different volume is given by $\Delta w_m = \frac{1}{2}\Delta L(\Delta I)^2$

$$\Delta w_m = \frac{1}{2}\Delta L(\Delta I)^2 \longrightarrow (3)$$

Sub eq (1) & (2) in eq (3) we get

$$\Delta w_m = \frac{1}{2}\frac{\mu H \Delta x \Delta z}{\Delta I}(H\Delta y)^2$$

$$= \frac{1}{2}\frac{\mu H \Delta x \Delta z}{\cancel{(H\Delta y)}}\cdot(H\Delta y)^{\cancel{2}}$$

$$= \frac{1}{2}(\mu H^2 \Delta x \Delta y \Delta z)$$

$$= \frac{1}{2}\mu H^2 \Delta x \Delta y \Delta z.$$

$$\because \Delta V = \Delta x \Delta y \Delta z$$

$$\boxed{\Delta w_m = \frac{1}{2}\mu H^2 \Delta V} \longrightarrow (4)$$

The magneto statics Energy density is defined as

$$w_m = \lim_{\Delta V \to 0} \frac{\Delta W_m}{\Delta V}$$

$$w_m = \lim_{\Delta V \to 0} \frac{\frac{1}{2}\mu H^2 \cancel{\Delta V}}{\cancel{\Delta V}}$$

$$w_m = \frac{1}{2}\mu H^2 \quad (\text{Joules}/m^3)$$

The magneto statics Energy density can be expressed in different forms

$$w_m = \frac{1}{2}(\mu H)H$$

$$w_m = \frac{1}{2}BH$$

$$w_m = \frac{1}{2} B\left(\frac{B}{\mu}\right)$$

$$B = \mu H$$

$$H = \frac{B}{\mu}$$

$$w_m = \frac{1}{2} \frac{B^2}{\mu}$$

$$\boxed{w_m = \frac{1}{2}\left(\frac{B^2}{\mu}\right)}$$

5. Time Varying field

Faradays Laws of Electro Magnetic induction:-

First law:-

Whenever a conductor cuts the Magnetic field or Magnetic flux lines then dynamically induced Emf in that conductors

Second law:-

~~Whenever~~ The dynamically induced emf is directely proportional to the rate of change of flux linkages

$$e \propto \frac{d\phi}{dt} \text{ volts}$$

$$e = -N\frac{d\phi}{dt}$$

Explanation:-

Let us considle a coil having a no. of turns 'N', and flux linkages with coil minimum ϕ_1 & Maximum ϕ_2 and the conductor in a time period t in seconds

The initial flux linkages $= N\phi_1$

The Final flux linkage $= N\phi_2$

Change of flux linkages ϕ = ~~$N\phi_2 - N\phi_1$~~ $N\phi_2 - N\phi_1$

$$\phi = N(\phi_2 - \phi_1)$$

Rate of change of flux linkages $= N\frac{d\phi}{dt}$

$$\therefore e = -N\frac{d\phi}{dt}$$

The Negative is due to lenz's law & indicates

The induced voltage is direction to the opposes to change in flux linkage that producing it

2 poynting vector & poynting theorem:-

* In ElectroMagnetic waves, an Energy can be transpo-rted from the Transmeter to the Recieuer.
* The Energy Stored in an Electricfield and Magnetic field Energy is transmitted at a certain rate of Energy flow which can be calculated with the help of poynting theorem.
* The product $\bar{E}$ & $\bar{H}$ gives the New quantity which is Expressed as watt/unit Area. Thus this Quantity is called power density
* Consider that the field is transmitted in the form of an Electromagnetic waves from an antenna, Both the fields are Senusoidal in Nature
* The Power Radiated from the Antenna as a parti-cular direction So the power density is given by

$$P = \bar{E} \times \bar{H}$$

where P is called poynting Vector

$\bar{E}$ is called Electric field

* pointing theorem is based on law of conservation of energy in Electro Magnetism.
* The net power flowing out of a given Volume (V) is equal to the rate of decrease in the

energy stored within volume V - The omic power despeated

[x] Since $\bar{E} = \bar{E_x} \cdot \bar{a_x}$

[x] $\bar{H} = \bar{H_y} \cdot \bar{a_y}$

[x] $\bar{P} = \bar{E} \times \bar{H}$

$\bar{P} = E_x \cdot a_x \times \bar{H_y} \cdot \bar{a_y}$

$\bar{P} = \bar{E_x} \bar{H_y} \bar{a_z}$ $\quad \because \bar{a_x} \times \bar{a_y} = \bar{a_z}$

$\bar{P} = \bar{P_z} \bar{a_z}$ $\quad \because \bar{P_z} = \bar{E_x} H_y$

[x] The above equation $\bar{E}, \bar{P}, \bar{H}$ are Mutually perpen-dicular to eachother.

[x] consider that the ~~di~~electric field propagates in free Space is given by $\bar{E} = E_m(\cos(\omega t - \beta z))\bar{a_x}$

[x] The Ratio of Magnitudes of $\bar{E}$ & $\bar{H}$ depends on its intrinsic impedence η for free Space

$$\eta = \eta_0 = \frac{\bar{E_m}}{H} = 120\pi = 377\,\Omega$$

[x] The Electromagnetic wave travels at speed of light in

$$\bar{H} = H_m(\cos(\omega t - \beta z))\bar{a_y}$$

[x] Since $H_m = \dfrac{E_m}{\eta_0}$

$$\bar{H} = \frac{E_m}{\eta_0}(\cos(\omega t - \beta z))\,\bar{a_y}$$

$$\bar{P} = \bar{E} \times \bar{H}$$

$$\bar{P} = E_m \cos(\omega t - \beta z)\, a_x \times \frac{E_m}{\eta_0}(\cos(\omega t - \beta z))\bar{a_y}$$

$$\bar{P} = \frac{E_m^2}{\eta_0} \cos(\omega t - \beta z)\,\bar{a_z} \,(\text{watt}/m^2)$$

3 Maxwell's equations for Time Variant fields:-

(a) Maxwell Eqn from faraday's law:-

consider faradays which relates Emf induced in a circuit to the time rate of decrease of total Magnetic flux linking the circuit.

$$\oint E \cdot \bar{dl} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot \bar{dS}$$

*] This equation is called as maxwell equation derived from faradays law Expressed in integral form

*] The Total Electro motive force induced in a closed path is Equal to the negative Surface integral of the rate of change of flux density w.r.t time over an entire Surface bounded by the same closed path.

$$\int_S (\Delta \times \bar{E}) \cdot \bar{dS} = -\int_S \frac{\partial \bar{B}}{\partial t} \cdot \bar{dS}$$

$$\Delta \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

This eqn is called maxwell equation derived from faradays Expressed in point from or differential form

(b) Maxwell equation derived from Amphere circuit law:-

According to Amphere circuit law the line integral Magnetic field intensity $\bar{H}$ around a closed path is equal to the current

Enclosed by $\oint \bar{H} \cdot \bar{dl} = I$ enclosed

Statement:

The Total magneto motive force around any closed path is equal to the surface integral of the conduction and displacement current densities over the entire surface bounded by the same closed path

$$\oint \bar{H} \cdot \bar{dl} = \int_S \bar{J} \cdot \bar{ds}$$

$$\boxed{\oint \bar{H} \cdot \bar{dl} = \int_S \left[\bar{J} + \frac{\partial \bar{D}}{\partial t}\right] \cdot \bar{ds}}$$

This Equation is called Maxwell Equation derived from Amphere circuit Law in integral form

$$\int_S (\Delta \times \bar{H}) \bar{ds} = \int_S \left[\bar{J} + \frac{\partial \bar{D}}{\partial t}\right] \cdot \bar{ds}$$

$$\boxed{\Delta \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}}$$

The equation is called as maxwell equation or point form or differential form derived from Ampheres ckt law

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